The most similar target based of inputs to the assessed unit

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Abstract: DEA is a nonparametric method of calculating the relative efficiency of a DMU and yielding a reference target for an inefficient DMU. However, it is very hard for inefficient DMUs to be efficient by benchmarking a target DMU which has different input use. Finding appropriate benchmarks based on the similarity of inputs makes it easier for an inefficient DMU to try to be like its target DMUs. But it is rare to discover a target DMU, which is both the most efficient and similar in inputs, in real situation. Therefore, it is necessary to find the most similar and closest real DMU in terms of inputs on the strong efficiency frontier, which has the highest possible output. So we proposed a model that is a combination of the Enhanced Russell model and the additive model for inefficient DMUs to improve their efficiency.

Keywords: DEA, DMU, Benchmarking, Closest target.

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric technique based on mathematical programming for the evaluation of technical efficiency of a set of decision making units (DMUs) that consumes inputs to produce outputs. The efficiency score is got from the distance between the evaluated DMU and a point on the frontier of the technology that assists as efficient target. Information on targets can express an important role since they show keys for inefficient units to improve their performance.

Traditional DEA models calculated efficient targets which have the furthest projection on the efficiency frontier. Therefore, a number of authors [6,14] discuss that the distance to the efficient projection point should be minimized, instead of maximized, until the resulting targets to be as similar as possible to the inputs and outputs of the evaluated unit. Because improving the inputs and outputs of the assessed unit relative to the closed target requires less effort than reaching the furthest target. Determining closest targets has been one of the essential subjects in the DEA literature and also the determination of closest targets is problematic enough and this fact justifies the work to apply new ways in order to overcome it.

In the meantime, the topic of finding the closest targets for DMUs to be efficient has involved growing interests of many researchers in DEA area. There are several methods for finding the closets targets because different researchers gave different definitions about the closest targets and also suggested different ways. Some of them minimized the selected distance and the other ones minimized the chosen efficiency measure. We can refer to some interesting articles: Frei and Harker [10] gave the closest targets by minimizing the Euclidean distance to the efficient frontier. Amirteimoori and Kordrostami [1] and Aparicio and Pastor [5] applied the weighted versions of the Euclidean distance to obtain the closest targets. Jahanshahloo et al, [12] proposed a technique for obtaining the minimum distance of DMUs from the frontier of the PPS by $\|\bullet\|_1$. Ando et al, [3] pointed out that least distance measures based on holder norms. Aparicio and Pastor [4] obtained a solution for output-oriented models based on an extended PPS that is strongly monotonic. Fukuyama et al, [11] calculated smallest distance p-norm inefficiency measures which satisfy strong monotonicity over the strongly efficient frontier to obtain the benchmarks. An et al, [2] combined enhanced Russell measure and closet targets to provide the closest targets. We will briefly explain their method in the background section of this paper.

In this paper, we have provided a new definition for the target of the evaluated DMU, so we proposed a model that is a combination of the Enhanced Russell model and the additive model.

2. Literature Review

2.1 The additive model

Assume that there are a set of n DMUs, and each $DMU_j, (j=1,2,\dots,n)$ produces s different outputs using m different inputs which are denoted as $y_{rj} (r=1,2,\dots,s)$ and $x_{ij} (i=1,2,\dots,m)$, respectively. Additive model which has been provided by Charnes et al. [7] to evaluate decision making units is defined as follows:

$$
\begin{aligned}
E_{VRS} = \min \quad & \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \quad i=1,\cdots,m, \qquad (1) \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r=1,\cdots,s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& s_i^- \geq 0, \quad i=1,\cdots,m, \\
& s_r^+ \geq 0, \quad r=1,\cdots,s, \\
& \lambda_j \geq 0, \quad j=1,\cdots,n.
\end{aligned}
$$

2.2 Enhanced Russell measure

Assume that there are a set of n DMUs, and each $DMU_j, (j=1,2,\dots,n)$ produces s different outputs using m different inputs which are denoted as $y_{rj} (r=1,2,\dots,s)$ and $x_{ij} (i=1,2,\dots,m)$, respectively.

The Russell measure of technical efficiency is a non-orientation efficiency measure that proposed by Fare and Lovell [9]. This model has computational and explanatory problems therefore Pastor et al. [13] constructed an enhanced measure model for measuring the efficiency, as follows:

$$
\begin{aligned}
E_{CRS} = \min \quad & \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \\
s.t \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{io}, \quad i=1,\cdots,m, \qquad (2) \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \ge \varphi_r y_{ro}, \quad r=1,\cdots,s, \\
& 0 \le \theta_i \le 1, \quad i=1,\cdots,m, \\
& \varphi_r \ge 1, \quad r=1,\cdots,s, \\
& \lambda_j \ge 0, \quad j=1,\cdots,n.
\end{aligned}
$$

E_{CRS} is the efficiency of DMU_O. If $E_{CRS}=1$, then DMU_O is Pareto efficient DMU also if $E_{CRS}<1$ then DMU_O is inefficient DMU. This model is under the assumption of constant return to scale (CRS), We can simply extend this model to non-decreasing (NDRS), non-increasing (NIRS) and variable return to scale (VRS) by addition $\sum_{j=1}^{n}\lambda_j \le 1, \sum_{j=1}^{n}\lambda_j \ge 1, \sum_{j=1}^{n}\lambda_j = 1$ in the constraints of model (1), respectively.

Model (2) can be easily changed into a same linear programming, shown as follows:

$$
\begin{aligned}
E_{CRS} = \min \quad & \frac{1}{m}\sum_{i=1}^{m}\Theta_i \\
s.t \quad & \sum_{r=1}^{s}\Phi_r = s, \\
& \sum_{j=1}^{n}\Lambda_j x_{ij} \le \Theta_i x_{io}, \quad i = 1,\cdots,m, \qquad (3) \\
& \sum_{j=1}^{n}\Lambda_j y_{rj} \ge \Phi_r y_{ro}, \quad r = 1,\cdots,s, \\
& 0 \le \Theta_i \le w, \quad i = 1,\cdots,m, \\
& \Phi_r \ge w, \quad r = 1,\cdots,s, \\
& \Lambda_j \ge 0, \quad j = 1,\cdots,n, \\
& 0 \le w \le 1.
\end{aligned}
$$

Note: If model (2) is in states non-decreasing (NDRS), non-increasing (NIRS) or variable return to scale (VRS), then constrain related to the type of return to the desired scale, i.e. $\sum_{j=1}^{n}\lambda_j \le 1, \sum_{j=1}^{n}\lambda_j \ge 1, \sum_{j=1}^{n}\lambda_j = 1$ will be converted to constrain $\sum_{j=1}^{n}\Lambda_j \le w, \sum_{j=1}^{n}\Lambda_j \ge w, \sum_{j=1}^{n}\Lambda_j = w$ in model (3), respectively.

2.3 Closet targets model based on enhanced Russell measure

An et al. [2] firstly constructed an enhanced Russell measure model in the existence of undesirable output, and then built the closet targets for the evaluated DMU_O under the enhanced Russell model. Therefore they showed the input, desirable output and undesirable output of DMU_j by x_j, y_j, z_j, respectively. Then their model for measuring the efficiency in the presence of the undesirable output obtained as follows:

$$
\begin{aligned}
E_{CRS} = \min \quad & \frac{1}{m}\sum_{i=1}^{m}\Theta_i + \frac{1}{p}\sum_{t=1}^{p}\phi_t \\
s.t \quad & \sum_{r=1}^{s}\Phi_r = s, \\
& \sum_{j=1}^{n}\Lambda_j x_{ij} \le \Theta_i x_{io}, \quad i=1,\cdots,m, \qquad (4) \\
& \sum_{j=1}^{n}\Lambda_j y_{rj} \ge \Phi_r y_{ro}, \quad r=1,\cdots,s, \\
& \sum_{j=1}^{n}\Lambda_j z_{tj} \le \phi_t z_{to}, \quad t=1,\cdots,p, \\
& 0 \le \Theta_i \le w, \quad i=1,\cdots,m, \\
& \Phi_r \ge w, \quad r=1,\cdots,s, \\
& 0 \le \phi_t \le w, \quad t=1,\cdots,p, \\
& \Lambda_j \ge 0, \quad j=1,\cdots,n, \\
& 0 \le w \le 1.
\end{aligned}
$$

E_{CRS} is the efficiency of DMU_O. If $E_{CRS}=1$, then DMU_O is Pareto efficient DMU also if $E_{CRS}<1$ then DMU_O is inefficient DMU. Unfortunately, the above model finds the maximum distance between the evaluated DMU and the efficient production frontier. So, they proposed the closest target method, to find the closest target for the evaluated DMUs.

Assuming that E is the set of efficient units of model (4), they proved under hypothesis constant returns to scale, any virtual and real DMU formed by the Russell efficient DMUs set E concluded $\sum_{j\in E}\Lambda_j x_j = X$, $\sum_{j\in E}\Lambda_j y_j = Y$, $\sum_{j\in E}\Lambda_j z_j = Z$ is efficient. Because of this theorem, they found the closet targets to the evaluated DMU_O by model (5):

$$
\begin{aligned}
E'_{CRS} = \max \quad & \frac{1}{m}\sum_{i=1}^{m}\Theta_i + \frac{1}{p}\sum_{t=1}^{p}\phi_t \\
s.t \quad & \sum_{r=1}^{s}\Phi_r = s, \\
& \sum_{j\in E}\Lambda_j x_{ij} \le \Theta_i x_{io}, \quad i=1,\cdots,m, \qquad (5) \\
& \sum_{j\in E}\Lambda_j y_{rj} \ge \Phi_r y_{ro}, \quad r=1,\cdots,s, \\
& \sum_{j\in E}\Lambda_j z_{tj} \le \phi_t z_{to}, \quad t=1,\cdots,p, \\
& 0 \le \Theta_i \le w, \quad i=1,\cdots,m, \\
& \Phi_r \ge w, \quad r=1,\cdots,s, \\
& 0 \le \phi_t \le w, \quad t=1,\cdots,p, \\
& \Lambda_j \ge 0, \quad j\in E, \\
& 0 \le w \le 1.
\end{aligned}
$$

They measured the optimal solution of model (4) as $(\theta^*, \Phi^*, \phi^*, \Lambda^*, w^*)$, then obtained the proportion of the inputs (α^*), the desirable outputs (Φ^*) and the undesirable outputs (ϕ^*) of DMU that be obtained by $\left(\theta^* = \Theta^*/w^*, \varphi^* = \Phi^*/w^*, \gamma^* = \phi^*/w^*\right)$. According to $(\theta^*, \varphi^*, \gamma^*)$, they could catch the closest targets for the inefficient DMUs to be efficient.

3. Research Findings

To determine the efficient DMUs in this work we consider the Enhanced Russell Measure model. One of the advantages of using enhanced Russell measure is because of that this model determines the strong efficient DMUs and gives us all DMUs which located on strong frontier. This model is unable to assign the weak efficient DMUs. So we use this model due to that weak efficient DMUs may be dominated by some DMUs and the benchmark can be introduced for them.

Therefore the aim of this work is to find a benchmark for DMUs that by enhanced Russell measure model are evaluated as inefficient DMUs with $E^{*}_{VRS} < 1$.

We consider smallest neighborhood around DMU under evaluation based on its inputs. So we find a real efficient DMU in this neighborhood which its inputs have smallest distance to DMU under evaluation's inputs, i.e. its inputs is could be as close to the inputs of the DMU under evaluation as possible, while it has the largest outputs. For example may be there are several efficient DMUs in this neighborhood but we only consider that DMU with largest and best outputs as a benchmark. Since we are looking for a benchmark's DMU among real DMUs not virtual DMUs, so the proposed model is a binary model as presented below:

$$
\begin{aligned}
\min \quad & \frac{1}{m}\sum_{i=1}^{m} s_i - \frac{1}{s}\sum_{r=1}^{s} \varphi_r \\
s.t. \quad & \sum_{j\in E} \lambda_j x_{ij} + s_i = x_{io}, \qquad i = 1,\dots,m, \qquad\qquad (6) \\
& \sum_{j\in E} \lambda_j y_{rj} \geq \varphi_r y_{ro}, \qquad r = 1,\dots,s, \\
& \sum_{j\in E} \lambda_j = 1, \\
& \lambda_j \in \{0,1\}.
\end{aligned}
$$

In which E is a set of efficient DMUs by obtaining from model (3) and s_i is a slack of input $i\ (i = 1,\dots,m)$. It means that, the minimum s_i is searched that can be deducted from inputs and maximum outputs that can be given to the DMU in a combination of real and efficient DMUs ($j \in E$).

Since, only one real benchmark is searched, the constraint $\lambda_j \in \{0,1\}$ must be considered. If we look for a virtual benchmark DMU, the constraint $\lambda_j \geq 0$ is used instead of $\lambda_j \in \{0,1\}$.

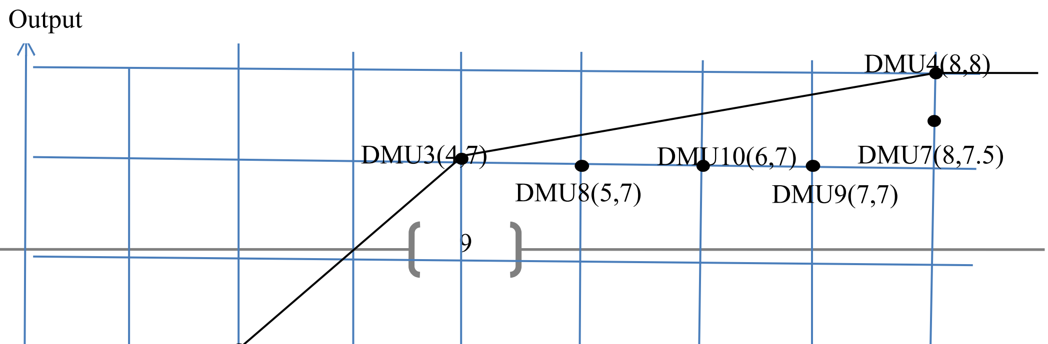
Important note: In solving model (6), we can substitute constraint $\lambda_j \geq 0$ instead of constraint $\lambda_j \in \{0,1\}$, then, in the optimal answer, if one DMU has λ^*_j with a value of one, $(\lambda^*_j = 1)$, then the same DMU is selected as the benchmark, if there are more than one DMU that have a positive λ^*_j $(\lambda^*_j > 0)$, then we choose the DMU that has the largest λ^*_j as a benchmark of the evaluated DMU.

4. Case Study

We consider 10 DMUs with one input and one output. The data of these DMUs are shown in Table 1. The Production Possibility Set (PPS) that these 10 DMUs are created, is shown in Figure1.

Table 1. Inputs and Outputs of 10 DMUs.

DMUs	Input	Output
DMU_1	1	2
DMU_2	2	5
DMU_3	4	7
DMU_4	8	8
DMU_5	4	4
DMU_6	2	3
DMU_7	8	7.5
DMU_8	5	7
DMU_9	7	7
DMU_{10}	6	7



As seen in Figure 1. DMU_1, DMU_2, DMU_3, DMU_4 are evaluated as efficient and others DMUs are inefficient. We want to calculate the benchmarks of these inefficient DMUs based on proposed model in this paper. The results of running model (5) are shown in Table 2.

Table 2. The results of determination Benchmarking from inefficient DMUs.

DMUs	$\lambda^*_j \geq 0 \ \ (j \in E)$	**Benchmark**
DMU_5	$\lambda_3 = 1$	DMU_3
DMU_6	$\lambda_2 = 1$	DMU_2
DMU_7	$\lambda_4 = 1$	DMU_4
DMU_8	$\lambda_3 = 0.75,\ \lambda_4 = 0.25$	DMU_3, DMU_4
DMU_9	$\lambda_3 = 0.25,\ \lambda_4 = 0.75$	DMU_3, DMU_4
DMU_{10}	$\lambda_3 = \lambda_4 = 0.5$	DMU_3, DMU_4

As seen in Table 2, the benchmark of DMU_5, is DMU_3. It means that the input of DMU_3 is similar to DMU_5 input, but it has better output rather than the output of DMU_5. So, in order to DMU_5 reach to an efficient status, there is no need to increase or decrease its input and With this input can reach to an efficient DMU. The DMU_5

must be compared with DMU_3 and accept it as a benchmark. Also DMU_5 should increase its output same as DMU_3.

DMU_2 and DMU_4 are as the benchmark for DMU_6 and DMU_7 with same inputs respectively. DMU_3, DMU_4 are as two benchmarks for DMU_8, DMU_9, and DMU_{10}.

As seen in Table 2, DMU_8 has $\lambda_3 = 0.75,\ \lambda_4 = 0.25$, it means that this DMU is %75 similar to DMU_3 and %25 similar to DMU_4. So, for DMU_8 is easy to choose DMU_3 as its real target, because input of DMU_8 is closer to input of DMU_3 rather than the input of DMU_4.

For DMU_9, $\lambda_3 = 0.25,\ \lambda_4 = 0.75$ which implies that the data of DMU_9 is closer to DMU_4 and reach to this DMU is easier in compare to DMU_3.

Finally DMU_{10} has same λ ($\lambda_3 = \lambda_4 = 0.5$).This company has more freedom of action and can move itself to the place of DMU_3 or DMU_4 to be efficient. In other words, he chooses whichever one is easier and more capable for him as a benchmark.

5. Conclusion

The main purpose of this study is to propose a method to obtainthe best benchmark for inefficient DMUs based on similarity in inputs. A benchmark that is selected from real DMUs, not virtual DMU that does not exist externally. It is rare to find out a target DMU with input endowments similar to that of an inefficient DMU. We use an enhanced measure model for measuring the efficiency DMUs because this model determines the strong efficient DMUs and gives us all DMUs which located on strong frontier. We consider smallest neighborhood around DMU under evaluation based on its inputs. So we find a real efficient DMU in this neighborhood which its inputs have smallest distance to DMU under evaluation's inputs, i.e. its inputs is could be as close to the inputs of the DMU under evaluation as possible, while it has the largest outputs. Therefore we proposed a model that is a combination of the Enhanced Russell model and the additive model.

We think that the target introduced by our proposed method is more practical target for the evaluated unit, and this inefficient unit can improve its efficiency more easily by this benchmark.

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