Super-Twisting Sliding Mode Control Design for a Class of Nonlinear Fractional Chaotic Systems

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Abstract

In this paper, a fractional super-twisting (FST) sliding mode controller design is derived for a class of nonlinear fractional chaotic systems. A new theorem which facilitates designing procedure of robust controller is provided. The stability of the closed loop system is verified in sense of the Lyapunov theorem. The main merits of the mentioned controller designing method are 1) stability of the closed loop system, 2) convergence of the output tracking error to zero, 3) robustness against external disturbance and uncertainty, and 4) reduction of chattering phenomenon. Finally, the simulation results demonstrate the capability and efficiency of the proposed controller methodology.

Keywords: Fractional-order system, Super Twisting algorithm, Sliding Mode Control, Lyapunov function.

1 Introduction

In recent year, fractional calculus has more attraction of the researchers [7]. About 300 years ago, Fractional calculus was the mathematical topic, but nowadays it develops rapidly in engineering [8]. The subject of fractional calculus has a lot of applications mainly in the field of mathematical sciences [9], and engineering such as electromagnetics, viscoelasticity, fluid mechanics, electrochemistry, biological population models, optics, and signals processing [2], chaos phenomena [10], controllers and observers design [15]. In order to model physical and engineering processes in which use the fractional differential equations because of its accuracy than the integer order dynamical systems for description and modeling of a real object [11]. In the past decades, among the investigations of fractional order systems control design for some fractional order systems has been a hot topic. Many different control methods have been proposed for various kinds of fractional systems [5, 13].

Sliding mode control technique for designing robust controller is common approach. The sliding mode control is the variable structure control, which have the simple overall structure, fast response, no sensitive about the internal parameters and external disturbances [1, 12]. However, the traditional sliding mode control has the chattering, because of the discontinuity nature of controller design. For reduction of the chattering and maintain the merits of traditional sliding mode, high order sliding mode is proposed in [4, 6]. The super twisting algorithm is one of the most applied methods in high order sliding mode.

Compare to the other researches, this paper focuses on the super twisting sliding mode controller design for disturbed nonlinear system. Our methodology has the following merits:

1- Deriving a fractional class of super twisting sliding mode controller to reduce the chattering phenomena.
2- The stability of the closed loop system and convergence of the tracking error to zero are both guaranteed in presence of the disturbances.

In this paper, section 2 includes basic definition of fractional integral and differential definition, section 3 provides a problem formulation, and in section 4, a new super twisting sliding mode controller for a class of nonlinear fractional systems introduces. In section 5, illustrates simulation results of the proposed method on chaotic fractional system. Finally, section 6 involves some brief conclusions.

2 PRELIMINARIES

In this section, some basic definitions of the fractional systems are given.

Fractional calculus is a generalization of integration and differentiation to non-integer-order fundamental operator. The continuous integro-differential operator is defined as:

$$ {}_aD_t^q = \begin{cases} \frac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \int_a^t (d\tau)^{-q} & q < 0 \end{cases} \qquad (1)$$

where *a* and *t* are the bounds of the operation and $q \epsilon$ R is the order [8].

There are three common definitions for fractional derivative and one for fractional integral, which describe in the below text.

Definition 2.1: The Grunwald - Letnikov (G), Riemann-Liouville (RL), and Caputo(C) derivative of order q of function f(t) is described as:

$$G \qquad {}_a^{GL}D_t^q f(t) = \lim_{N\to\infty} \left[\frac{t-a}{N}\right]^{-q} \sum_{j=0}^{N-1} (-1)^j \binom{q}{j} f\left(t - j\left[\frac{t-a}{N}\right]\right) \qquad (2)$$

$$RL \qquad {}_a^{RL}D_t^q f(t) = \frac{1}{\Gamma(1-q)} \frac{d}{dt} \int_a^t (t-\tau)^{-q} f(\tau) d\tau \qquad (3)$$

$$C \qquad {}_a^{C}D_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_a^t (t-\tau)^{-q} \dot{f}(\tau) d\tau \qquad (4)$$

Where $0 < q < 1$ and $\Gamma(.)$ is the Gamma function.

Definition 2.2: The Riemann-Liouville fractional integral of order q is defined as:

$$ {}_aD_t^{-q} f(t) = \frac{1}{\Gamma(q)} \int_a^t (t-\tau)^{q-1} f(\tau) d\tau \qquad (5).$$

3 PROBLEM FORMULATION

In this section, the super twisting sliding mode controller is designed for a wide class of nonlinear fractional order systems as:

$$\begin{cases} D^q x = F_1(x,y,z) \\ D^q y = F_2(x,y,z) + u(t) + \sigma(t) \\ D^q z = F_3(x,y,z) \end{cases} \qquad (6)$$

where x=$[x, y, z]^T$ is the state variable, F_1 , F_2 and F_3 show nonlinear function ,and u presents the control input and furthermore $\sigma(t)$ shows the uncertainties. The Chen, Liu, Lu, and Lorenz chaotic systems can be presented in form of the equation (6).

Conventional sliding mode controller design uses linear sliding-mode (LSM) surface to track the desire trajectory, this causes inherently nonlinear control. The main challenge in LSM controller design is sliding surface selection based on the system performance requirements.

The first step to controller design is assigned a sliding surface such as:

$$s = x + \lambda_1 y + \lambda_2 z. \qquad (7)$$

where λ_1 and λ_2 are positive constants. Taking the q-order fractional differential of the above equation yields.

$$D^q s = D^q x + \lambda_1 D^q y + \lambda_2 D^q z \qquad (8)$$

The control objectives are 1) tracking of the desire trajectories, 2) convergence of the sliding surface to zero without chattering and 3) stability of the closed loop system. Without loss of generality, it is assumes the desire trajectories are zero.

4 SUPER-TWISTING SLIDING MODE CONTROL DESIGN FOR A CLASS OF NONLINEAR FRACTIONAL SYSTEMS

In last section, it has been shown the sliding surface to achieve control objectives. We show how to derive the control input. The total input of the system can be proposed as below.

$$u(t) = u_{eq}(t) + u_r(t) \qquad (9)$$

where the $u_{eq}(t)$ *and* $u_r(t)$ are the equal and the reaching controller parts that define as follows:

$$u_{eq}(t) = \frac{1}{\lambda_1}[-F_1(x,y,z) - \lambda_1\, F_2(x,y,z) - \lambda_2\, F_3(x,y,z) - \lambda_1\, \sigma(t)] \quad (10)$$

and

$$u_r(t) = \frac{1}{\lambda_1}[-\alpha|s|^{\rho} sgn(s) - \beta \int sgn(s)\, dt] \qquad (11)$$

The first term compensates the uncertainties of the model and the second one reduces the chattering phenomena in which the parameters mentioned in equation (11) satisfy the following inequalities.

$$0 < \rho < 1 \quad , \quad \alpha, \beta > 0$$

To facilitate the super twisting sliding mode controller, the below theorem is derived by the authors.

Theorem 4.1: consider the nonlinear fractional-order dynamical system is given in (6), and the controller structure proposed in (9), (10) and (11) based on the sliding surface mentioned in (7) make the closed loop system stable in sense of the Lyapunov, and furthermore, both the convergence of the error trajectory to zero and boundedness of all signals involved in the closed loop system are guaranteed.

Proof: we candidate the following Lyapunov function to investigate the closed loop stability.

$$V = \frac{1}{2}s^2 \qquad (12)$$

By taking the q-order fractional derivative of proposed Lyapunov function to produce.

$$D^q\mathrm{V} = \sum_{i=0}^{\infty}\binom{q}{i}s^{(i)}.D^{(q-i)}s < s.D^q s \qquad (13)$$

So, the above equation is rewritten as follow:

$$D^q\mathrm{V} < s.D^q s = s[D^q x + \lambda_1 D^q y + \lambda_2 D^q z] = s[F_1(x,y,z) + \lambda_1\, F_2(x,y,z) + \lambda_1\, \sigma(t) + \lambda_1 u + \lambda_2\, F_3(x,y,z)]\ . \qquad (14)$$

According to (9), the (14) can be rewritten as:

$$D^q\mathrm{V} < -\alpha|s|^{\rho+1} - \beta\int|s| < 0 \qquad (15)$$

According to standard Lyapunov theorem, we conclude the closed loop system is stable and accordingly the sliding surface converges to zero. In addition, the boundedness of the signals in the closed loop system is assured. It completes the proof.

To show the capability of the proposed controller, the mentioned method is applied on the class of chaotic system.

5 SIMULATION RESULTS

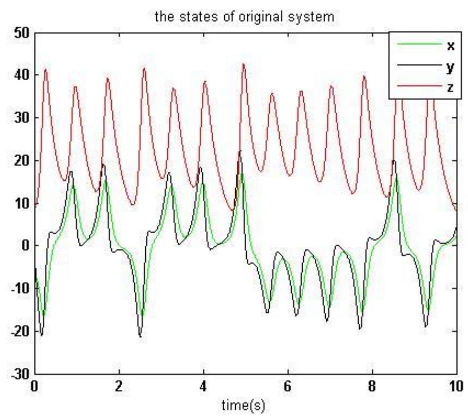
In this section, the proposed control scheme has been applied on the simulation test case in order to investigate the efficacies and capability.

Consider the following nonlinear q-order fractional Lorenz system.

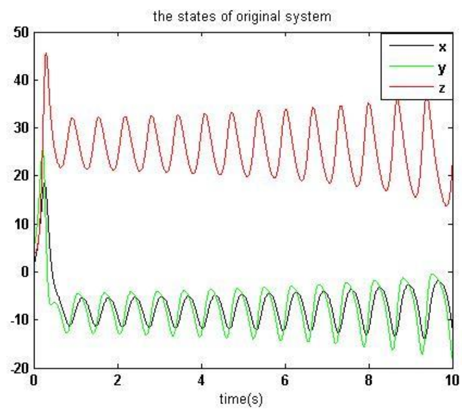
$$\begin{cases} D^q x = a(y-x) \\ D^q y = cx - xz - y + u(t) + \sigma(t) \\ D^q z = xy - bz \end{cases} \qquad (16)$$

Where $(a,b,c) = (10,8/3,28)$ and $\sigma(t) = \sin(wt)$ and be considered as external disturbance.

To show the extremely chaotic performance of the mention equation (16), the figure 1 is derived for the initial conditions$[\mathrm{x}(0),\mathrm{y}(0),z(0)] = [-9,-1,9]$ and $[\mathrm{x}(0),\mathrm{y}(0),z(0)] = [1,5,4]$.



(a)



(b)

Figure1. The state variable of original system: (a) with $[\mathrm{x}(0),\mathrm{y}(0),z(0)] = [-9,-1,9]$ and (b) with $[\mathrm{x}(0),\mathrm{y}(0),z(0)] = [1,5,4]$

Using the proposed controller in equation (9) the states of the system are shown in Figure 2.

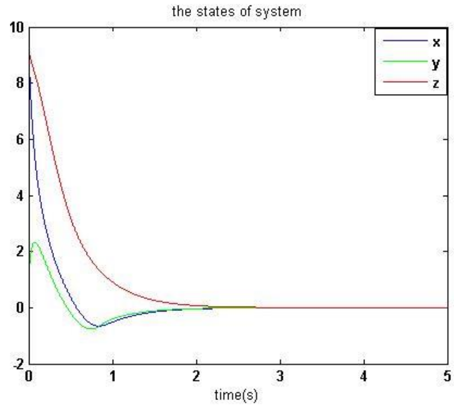
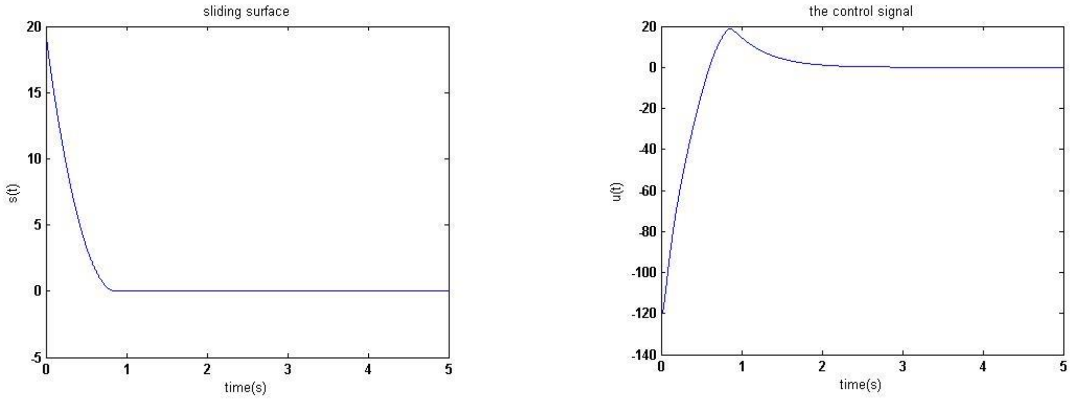


Figure2. The state variable with the proposed method with $[x(0), y(0), z(0)] = [9,1,9]$

According to the results of simulation, the comparison of Figures 1 and 2, it shows that the states of system asymptotically converge to zero without chattering.



(a) (b)

Figure3: (a) the proposed sliding surface and (b) the proposed control signal

Figure3a demonstrates the smoothness of the sliding surface tends to zero.

Figure3b shows that the control effort without any chattering phenomenon.

The simulation results show promising performances in both tracking and stability. It is obvious the proposed controller succeeds in chattering reduction.

6 CONCLUSION

This paper deals with a super-twisting sliding mode controller design for a class of chaotic nonlinear fractional order systems. Theoretical analysis by candidate Lyapunov function has been presented to show both the boundedness of all signals involved in closed loop system and stability of the closed loop system. Robustness of the proposed controller in presence of the external disturbances and furthermore the convergence of tracking error to zero are the main merits of the proposed controller design procedure. Simulation results authorize the promising performance of the proposed scheme.

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