On the Effects of Stiffness Ratio in Nonlinear Aeroelastic Stability of a Folding Wing

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Abstract
In this paper, the effects of stiffness ratio in nonlinear aeroelastic stability of a folding wing by using the geometrically exact fully intrinsic beam equations are investigated. The important advantages of these equations in comparison with other structural beam equations are complete modeling without simplifying assumptions in large deformations, low-order nonlinearities, and thus less complexity. For the first time, folding angles have been implemented in the geometrically exact fully intrinsic beam equations and hence this is the main novelty of this study. The applied aerodynamic loads in an incompressible flow regime are determined using Peter's unsteady aerodynamic model. In order to check the stability of the system, first the resulting non-linear partial differential equations are discretized, and then linearized about the nonlinear steady-state condition. By obtaining the eigenvalues of the linearized system, the stability of the wing is evaluated. Furthermore, investigation of the effects of the stiffness on the flutter speed of the folding wing for various folding angles, is another achievement of this study. It is observed that the geometrically exact fully intrinsic beam equations can model the folding angles for the aeroelastic analysis more accurately.
Keywords: *Folding wing-nonlinear aeroelasticity-fully intrinsic equations-folding angle.*

1. Introduction

The possibility of planform changes in the airplane wing turns it into a morphing wing. One of the states of changing the wing planform (extension, folding, sweep) is folding, which can improve the performance of the bird by adjusting the angles, size and related properties.

Many researchers have analyzed the dynamic behavior of the folding wings. One of the most prominent works in the aeroelastic study of folding wings is the research of Lee and Chen [1]. In their study, the nonlinear aeroelastic characteristics of a folding wing with free-play or piecewise nonlinearities at the inboard and outboard hinges are investigated. Ameri, et al. [2] presented a simulink multibody model for active winglets. The results of this research have shown a non-negligible dependency of the dynamic transient behavior on the shape variation. Tang and Dowell [3] used a model for theoretical and experimental aeroelastic study of folding wings. This study was then extended by combining the von Karman strain theory with three-dimensional vortex lattice aerodynamic model [4] to investigate the limit cycle oscillations of folding wings. Liska and Dowell [5] developed an analytical solution methodology to find the flutter speed of a two-segment uniform folding wing. In this research, different aerodynamic models were studied and compared, and the effect of each on flutter speed and folding angle was examined. Zhao and Hu [6] studied the flutter characteristics of a folding wing based on the substructure synthesis and aerodynamic doublet lattice method. This study demonstrated that the flutter characteristics of the folding wing are very sensitive to the folding angle. Ajaj, et al. [7] investigated the aeroelasticity of cantilever wings equipped with flared hinge folding wingtips. In this study, the wing structure is modelled employing Euler–Bernoulli beam theory, and unsteady Theodorsen's aerodynamic theory is used for aerodynamic load predictions. Also, the influence of folding angle, hinge-line angle, hinge stiffness, tip mass and nonlinear behavior of hinge are studied.

The present study is based on the geometrically exact fully intrinsic beam theory which was initially developed by Hegemier and Nair [8] and then revised by Hodges [9]. The important advantages of this theory in comparison with other structural beam theories are complete modeling without simplifying assumptions in large deformations, low-order nonlinearities, and thus less complexity. This beam theory has been employed by several researchers to investigate the wing aeroelastic stability. For example, Sotoudeh and Hodges [10] Investigated the effects of the sweep angle and position of joint on the dynamic stability and static deformation of a wing subjected to a follower force. They showed that the dynamic and static stability of the joint-wing is completely different from conventional wings. Moravej Barzani, et al. [11] studied the aeroelastic stability of swept wings employing geometrically exact beam formulation. The results showed that by utilizing the fully intrinsic beam equations, the aeroelastic instability of the swept wings can be determined more accurately. Amoozgar, et al. [12] studied the aeroelastic stability of a wing by considering the effect of bend-twist elastic coupling and taper ratio of wing in combination with the pre-twist angle. Moravej Barzani, et al. [13] investigated the effect of structural nonlinearity on the aeroelasticity of span morphing wings using the exact fully intrinsic

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equations and Peter's unsteady aerodynamic model. They showed that the morphing length and overlapping mass of the fixed and the moving segments have significant effects on the aeroelastic stability of telescopic wings.

In this study, for the first time, folding angles have been implemented in the geometrically exact fully intrinsic beam equations[9] to check the aeroelastic stability of folding wings. Investigation of the effects of some important parameters such as stiffness ratio on flutter speed of a folding wing for various folding angles, is one of the other achievements of this article which has not been given much attention in previous researches.

2. Governing equations

A schematic of a folding wing is shown in Fig. 1. The wing comprises three parts that have changed surface with folding angles θ_1 and θ_2.

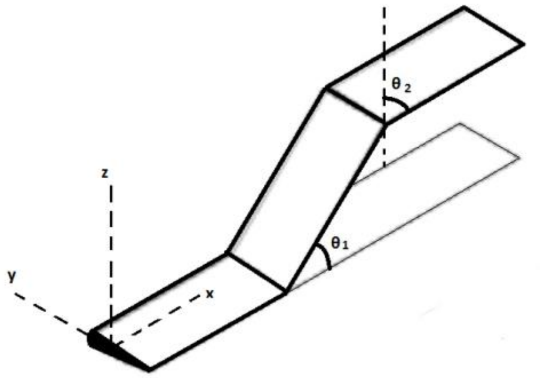

Fig. 1 a schematic of a folding wing

In this study, the geometrically exact fully intrinsic equations are employed to simulated the wing nonlinear dynamic behavior.

The geometrically exact fully intrinsic equations can be expressed as [9]

$$\begin{aligned} &F'_B + \widetilde{K}_B F_B + f_B = \dot{P}_B + \widetilde{\Omega}_B P_B \\ &M'_B + \widetilde{K}_B M_B + (\widetilde{e}_1 + \widetilde{\gamma})\Omega_B + m_B \\ &\quad = \dot{H}_B + \widetilde{\Omega}_B H_B + \widetilde{V}_B P_B \\ &V'_B + \widetilde{K}_B V_B + (\widetilde{e}_1 + \widetilde{\gamma})\Omega_B = \dot{\gamma} \\ &\Omega'_B + \widetilde{K}_B \Omega_B = \dot{k} \end{aligned} \tag{1}$$

where $\dot{(\,)}$ denotes the absolute time derivative, $(\,)'$ denotes the derivative with respect to the beam reference line and, M_B and F_B are the internal moment and force measures, P_B and H_B are the linear and sectional angular momenta, and κ and γ denote the moment and force strain measures, Ω_B and V_B are the angular and linear velocity measures. Also, $K_B = k_b + \kappa$ is the beam curvature vector in which k_b is the initial twist and curvature. Furthermore, m_B and f_B are the external moments and forces such as aerodynamic moments and forces.

By using the cross-section stiffness matrix, the generalized strains (γ, κ) and the generalized forces (F_B, M_B) are related together as follows

$$\begin{Bmatrix} \gamma \\ k \end{Bmatrix} = \begin{bmatrix} R & S \\ S^T & T \end{bmatrix} \begin{Bmatrix} F_B \\ M_B \end{Bmatrix} \tag{2}$$

where, S, T and R are the cross-sectional flexibility matrices.

Furthermore, the generalized moments (P, H) and the generalized velocities (V, Ω) can be converted to each other using the cross-sectional inertia matrix as

$$\begin{Bmatrix} P_B \\ H_B \end{Bmatrix} = \begin{bmatrix} \mu\Delta & -\mu\widetilde{\xi} \\ \mu\widetilde{\xi} & I \end{bmatrix} \begin{Bmatrix} V_B \\ \Omega_B \end{Bmatrix} \tag{3}$$

Where μ is the mass per unit length, I is the inertia matrix per unit length, ξ is the cross-sectional mass centroid offset from the beam reference axis and Δ is the identity matrix.

By using Peters' unsteady aerodynamic model, the Wing aerodynamic loads can be expressed as[14]

$$f_a^n = \rho b^n \begin{Bmatrix} f_{a1}^n \\ f_{a2}^n \\ f_{a3}^n \end{Bmatrix}$$

$$m_a^n = 2\rho b^{n^2} \begin{Bmatrix} m_{a1}^n \\ m_{a2}^n \\ m_{a3}^n \end{Bmatrix}$$

$$\begin{aligned} &f_{a1}^n = 0 \\ &f_{a2}^n = -(C_{l_0}^n + C_{l_\beta}^n \beta^n) V_T^n V_{a_3}^n + C_{l_\alpha}^n (V_{a_3}^n + \lambda_0^n)^2 - C_{d_0}^n V_T^n V_{a_2}^n \\ &f_{a3}^n = (C_{l_0}^n + C_{l_\beta}^n \beta^n) V_T^n V_{a_3}^n - C_{l_\alpha}^n \dot{V}_{a_3}^n b/2 - \\ &\qquad - C_{l_\alpha}^n V_{a_2}^n (V_{a_3}^n + \lambda_0^n - \Omega_{a_1}^n b^n/2) - C_{d_0}^n V_T^n V_{a_3}^n \\ &m_{a1}^n = (C_{m_0}^n + C_{m_\beta}^n \beta^n) V_T^{n^2} - C_{m_\alpha}^n V_T^n V_{a_3}^n - b^n C_{l_\alpha}^n / 8 V_{a_2}^n \Omega_{a_1}^n - \\ &\qquad - b^{n^2} C_{l_\alpha}^n \dot{\Omega}_{a_1}^n / 32 + b^n C_{l_\alpha}^n \dot{V}_{a_3}^n \\ &m_{a2}^n = 0 \\ &m_{a3}^n = 0 \end{aligned}$$

$$\lambda_0^n = \frac{1}{2}\{b_{\inf low}\}^T \{\lambda^n\}$$

$$[A_{\inf low}]\{\dot{\lambda}^n\} + \left(\frac{V_T^n}{b^n}\right)\{\lambda^n\} = \left(-\dot{V}_{a_3}^n + \frac{b^n}{2}\dot{\Omega}_{a_1}^n\right)\{c_{\inf low}\} \tag{4}$$

where b is the semichord, a is the aerodynamic reference axis, and ρ is the air density. Also, λ is a column matrix which includes inflow states, and $[A_{\text{inflow}}]$, $\{b_{\text{inflow}}\}$, $\{c_{\text{inflow}}\}$ are constant matrices derived in Peters, et al. [15] work. Also,

$$\overline{V}_a^n = C_a^{n\,T} \overline{V}^n - \widetilde{y}_{mc}^n C_a^{n\,T} \overline{\Omega}^n$$

$$\overline{\Omega}_a^n = C_a^{n\,T} \overline{\Omega}^n$$

$$V_T = \sqrt{V_{a_2}^2 + V_{a_3}^2}$$

$$\sin\alpha = \frac{-V_{a_3}}{V_T} \tag{5}$$

C is the rotation matrix with the superscripts indicating the two reference frames in between which it transforms. [16]
and y_{mc} is a row matrix (position vector) from the beam reference axis to the midchord and can be written as

$$y_{mc}^n = [0 \quad \bar{y}_{ac}^n - \frac{b^n}{2} \quad 0] \tag{6}$$

3. Solution methodology

For solving the governing equations, a finite difference discretizing scheme is employed (as seen in the work of Chang [14]). To determine the stability of the wing, first the nonlinear steady-state of the system is obtained by dropping all time-dependent variable, and solving the resulting nonlinear equation by employing the Newton–Raphson method. Then, the wing aeroelastic stability is sought by investigating the eigenvalues of this linearized system. The resulting discretized equations can be rewritten in a compact form as

$$[\mathbf{A}]\{\dot{\mathbf{X}}\} + [\mathbf{B}(\mathbf{X})]\{\mathbf{X}\} = 0 \tag{7}$$

where $\{\mathbf{X}\}$ is a vector contains the aerodynamic and structural states.

4. Results and discussion

To check the validity of the developed aeroelastic model, first the flutter speed and frequency of Goland wing is obtained and compared with those reported in the literature. The properties of this wing are presented in Table 1. According to Table 2, It is clear that the present results are in a very good agreement with those presented by Patil [17] with a maximum difference of 0.3%.

Table 1: The properties of Goland wing

Parameter	Value
l(m)	6.1
c(m)	1.83
m(kg/m)	35.7
$EI_{\text{spanwise}}(\text{Nm}^2)$	9.765 x 10^6
$GJ(\text{Nm}^2)$	9.89 x 10^5
Spanwise elastic axis	33% chord
Center of gravity	43% chord
I(50% chord)(kg. m)	8.64

Table 2: The comparison of the flutter speed and frequency

	Patil (1999)	Present	Difference (%)
u_f(m/s)	135.64	136.06	0.3
ω_f(rad/s)	70.2	70.09	0.2

It should be noted that $v_f = \frac{u_f}{b\omega_\alpha}$ denotes reduced flutter speed, where ω_α is the first uncoupled torsional frequency. Also, the following nondimensional parameter is used

$$\psi = \frac{EI}{GJ} \tag{8}$$

In the first step, the effects of changing the stiffness ratio of the first part and changing the folding angle, on the reduced flutter speed of Goland wing is determined and shown in Fig. 2. This figure shows that at a specific value of the stiffness ratio, at the same time as the folding angle increases, the reduced flutter speed has decreased to the angle range of 30-40 degrees. Then it suddenly increases and reaches an optimal value in the range of 40-50 degrees. In the following, it will have a downward trend up to about 80 degrees and the highest value at an angle of 90 degrees.
Paying attention to the influence of the stiffness ratio shows that the stiffness ratio had an inverse effect on the reduced flutter speed (inverse effect with bending stiffness and direct effect with torsional stiffness); Of course, compared to the reference value, the trend of changes in the results in the range of 20 to 50 degrees is not the same. The results show that by reducing the stiffness ratio, in addition to increasing the reduced flutter speed, the corresponding folding angle has also increased up to 10 degrees, but the increase in this ratio only caused a negative effect and did not cause any specific change in the folding angle.

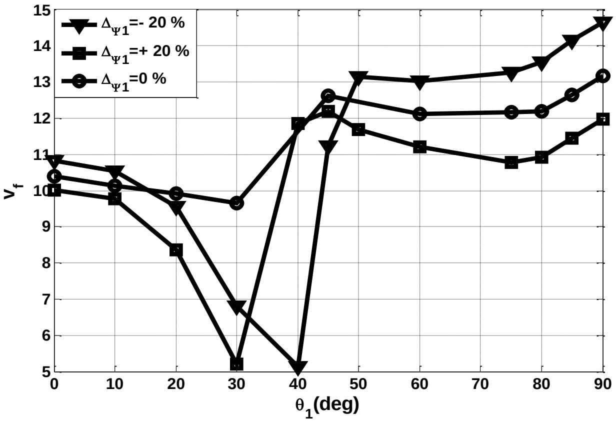

Fig. 2 Reduced Flutter speed vs the folding angle fordifferent values of the stiffness ratio of the first part

The effects of changing the stiffness ratio of the second part and changing the folding angle, on the reduced flutter speed is determined and shown in Fig. 3.

According to the figure, it can be seen that the trend of the changes in the results is consistent with the case in which the first part of the wing was investigated. On the other hand, the lower effect of changing the stiffness ratio of the second part of the wing compared to the first section has caused a lower effect on the reduced flutter speed, and this lower effect has also changed the folding angle up to 10 degrees in some points.

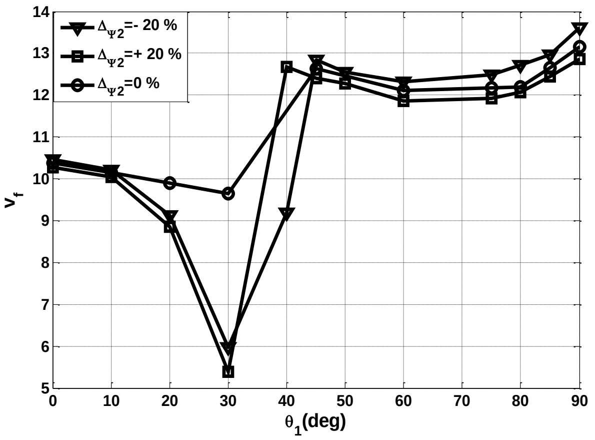

Fig. 3 Reduced Flutter speed vs the folding angle for different values of the stiffness ratio of the second part

The effects of changing the stiffness ratio of the third part and changing the folding angle, on the reduced flutter speed is shown in Fig. 4. This figure shows that changing the stiffness ratio of the third part has a small effect (about 2%) on the reduced flutter speed. Meanwhile, the trend of changes in the range of 10 to 40 degrees is significant compared to the normal case, and it indicates that in some folding angles, the effect of changing the stiffness ratio of the third part can be important. Also, by comparing the results obtained for other parts, it can be concluded that the stiffness ratio of the first part has a more important effect on the flutter stability than other parts.

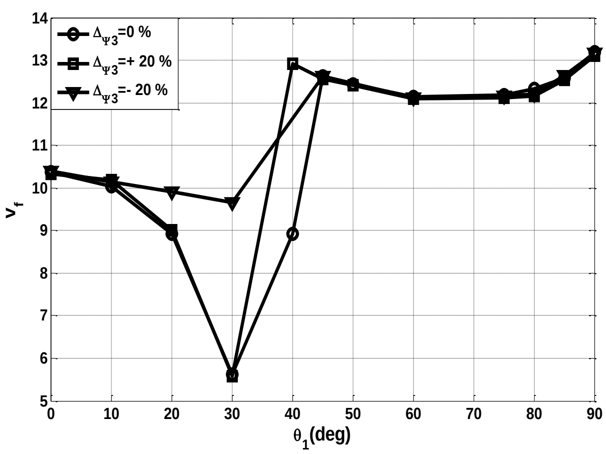

Fig. 4 Reduced Flutter speed vs the folding angle for different values of the stiffness ratio of the third part

5. Concluding Remarks

In this study, the effects of stiffness ratio in nonlinear aeroelastic stability of a folding wing by using the geometrically exact fully intrinsic beam equations were investigated. For structural and aerodynamic modeling, the geometrically exact fully intrinsic beam equations and Peters' unsteady aerodynamic model were used, respectively. The stability of the system with different parameters was evaluated by investigating the eigenvalues of the linearized system. The obtained results were compared with those available in the literature, and a good agreement was observed. Moreover, the effects of stiffness ratio on the reduced flutter speed of the folding wing for various folding angles were studied. The results of this study are summarized as follows:

1) The geometrically exact fully intrinsic beam equations can model the folding angles for the aeroelastic analysis and bring good results. The implemented formulation can be used to model the wing in different structures.

2) The stiffness ratio has significant effect on the aeroelastic stability of the folding wing.

3) The stiffness ratio of the first part has a more important effect on the flutter stability than other parts.

4) By properly selecting each of the parameters, the flight performance of the folding wing can be improved.

6. References

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