Design and Comparison of MPC and LQR Control Methods for a Passenger Aircraft

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Abstract
The paper compares the performance of two altitude controllers, MPC and LQR, for aircraft in cruise flight conditions. The design of the controllers is based on the linearized state space matrix of the aircraft's longitudinal motion around the trim conditions. The controllers' ability to track the desired altitude while satisfying input and state constraints is evaluated, and it is found that both controllers are effective in maintaining the desired altitude. However, the MPC controller outperforms the LQR controller in terms of limited control input, achieving smoother and more efficient control input by predicting the future behavior of the system. The proposed altitude controllers provide a promising solution for maintaining the desired altitude of aircraft in cruise flight conditions, and the comparative analysis of the two control methods can assist in the selection of the appropriate control strategy for a given aircraft system based on the desired performance requirements.

Keywords: *MPC, LQR, Fixed-wing, Aircraft, Altitude control*

1. Introduction

With the development of the air transportation industry and the increasing use of flying devices to carry passengers, the use of planes with less passenger capacity is expanding; jet planes are also a category of this group that usually has more agility and maneuverability [1]. The design of the control system determines whether there is sufficient knowledge of the dynamics and operation of the flying device. As we know, in real conditions, the relationships governing the dynamics of a flying vehicle are non-linear and variable with time. On the other hand, we are faced with problems such as the existence of un-modelled dynamics, the coupling of the equations governing the flying vehicle, and the ambiguity of the effect of aerodynamic coefficients on parameters such as the angle of attack, the angle of side slip, etc. If there is sufficient knowledge of the system, despite the mentioned problems, it is possible to use the basics of linear control or to use methods that are generalizations of linear control methods[2],[3],[4]. Flight control systems for passenger aircraft are still predominantly designed using classical control techniques. However, in recent years, modern methods have found more and more applications[5],[6]. The current flight control systems provide augmented stability and control. However, in the case of severe or unforeseen failures or changes in aircraft behavior (e.g. due to icing), the control system reverts to reversionary modes or even direct control, This implies that the control law functionality is partly reduced or abandoned. This behavior is undesirable, as the pilot's workload is not only increased due to failure but also due to aircraft control. Developments subsequently focus on maintaining functionality, even in the case of such failures. Research in this field includes reliable fault detection and diagnosis, and control reconfiguration[7],[8]. One aspect is the adaptation of flight control laws to unforeseen circumstances and failures[9]. Although model predictive controllers provide many advantages over classical or modern control methods (such as PID or LQR), their practical applications have been limited to high-level path planning, guidance logic, and control of slow robots with less complex dynamics[10]. In the paper [11], a robust MPC-based autopilot was designed for a mini UAV and key features of the proposed technique were: (i) the control gain matrix is evaluated offline to guarantee the real-time feasibility of the MPC; (ii) the controller is robust to parametric model uncertainties (i.e. mass and inertia variations) and to random bounded noise (i.e. gust). Lateral control during aircraft-on-ground deceleration is discussed in [12] based on linear, quadratic, and predictive control theories. One of the problems in passenger planes is the control of the plane in flight maneuvers.

In this paper, the dynamic model of a passenger plane was derived using Euler's equations. The nonlinear model was subsequently linearized around the cruise flight, and two controllers, namely Model Predictive Control (MPC) and Linear Quadratic Regulator (LQR), were compared for executing the desired maneuver. The selection of these controllers was based on the use of quadratic optimization in their control laws, with the primary difference lying in the application of online constraints and model prediction in MPC.
The outline for this paper is as follows: In Section II, we will deal with the mathematical modelling of the Cessna Citation II (550) aircraft, in Section III, the controllers are described, and in the last section, the results will be investigated and compared.

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2. Mathematical modelling

The control laws developed in this research are evaluated in the Cessna Citation II PH-LAB laboratory aircraft Fig[1] and a corresponding aerodynamic aircraft model coefficient which is derived from flight test data[13]. An aerodynamic derivative derived from [14] flight test vehicle dimension and mass properties can be found in Table 1.

Fig. 1:Cessna citation II.[9]

Table 1: PH_LAB dimension and mass properties.[13]

dimension	
b	15.9m
$\bar{c}$	2.09m
S	$30m^2$
Mass and inertia	
m	4,157kg
U_0	$225\frac{m}{s}$
I_{xx}	$12392kg.m^2$
I_{yy}	$31501kg.m^2$
I_{ZZ}	$41908kg.m^2$
I_{xz}	$2252.2kg.m^2$

The dynamic equations of the aircraft are sub-extracted:

$$\dot{u} = X_u u + X_w w - gcos\theta_0\theta + X_{\delta e}\delta e \quad [1]$$
$$\dot{w} = Z_u u + Z_w w + U_0 q - gsin\theta_0\theta + Z_{\delta e}\delta e \quad [2]$$
$$\dot{q} = M_u u + M_w w + M_q q + M_{\delta e}\delta e \quad [3]$$
$$\dot{\theta} = q \quad [4]$$
$$\dot{h} = -w + U_0\theta \quad [5]$$

which is the relationship above g is the earth's gravity, U_0 is the initial speed of the plane, u is the horizontal speed, w is the vertical speed, the angular speed around the longitudinal axis is q, the pitch angle θ, $X_u$stability derivatives in the direction of horizontal body axis relative to horizontal speed, $X_w$stability derivatives in the direction of horizontal body axis relative to vertical speed, X_{δ_e}control derivatives in the direction of horizontal body axis relative to elevator deflection, $Z_u$stability derivatives in the direction of vertical body axis relative to horizontal speed, $Z_w$stability derivatives in the direction of vertical body axis relative to vertical speed, Z_{δ_e}control derivatives in the direction of vertical body axis relative to elevator deflection, $M_u$represents the change in pitching moment caused by a change in forward speed, $M_w$is the change in pitching moment caused by a change in vertical speed, $M_q$is the change in pitching moment caused by a change in the rate of pitch angle, M_{δ_e} is the change in pitching moment caused by a change in elevator deflection. Table 2.

$$A = \begin{bmatrix} X_u & X_\omega & 0 & -gcos\theta_0 & 0 \\ Z_u & Z_\omega & U_0 & -gsin\theta_0 & 0 \\ M_u & M_\omega & M_q & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & U_0 & 0 \end{bmatrix}, B = \begin{bmatrix} X_{\delta e} \\ Z_{\delta e} \\ M_{\delta e} \\ 0 \\ 0 \end{bmatrix} \quad [6]$$

Table 2: stability and control coefficient.[13]

Stability & control coefficient			
X_u	-0.0072	Z_{δ_e}	0
X_w	0.1532	M_u	0
X_{δ_e}	15.8181	M_w	-0.0593
Z_u	-0.6753	M_q	-1.3017
Z_w	-1.7716	M_{δ_e}	-24.0336

3. Flight control design

3.1 LQR Controller

The goal of this controller is to find a control input that minimizes a cost function in the following form

$$J = \frac{1}{2}\int_{t_0}^{t_f} [(x(t) - x_d(t))^T(t)Q(x(t) - x_d(t)) + u_c^T(t)Ru_c(t)]dt \quad [7]$$

$$\dot{x}(t) = Ax(t) + Bu_c(t) \quad [8]$$

Above, x(t) is the state vector and equal to $[u \quad w \quad q \quad \theta \quad h]$, $u_c(t)$ is the input control matrix and equal to δe , A is the system state matrix, B is the control matrix, and R and Q are positive definite weight matrices.

As a result, the control law will be written as follows.

$$u_c(t) = -K\left[\left(x(t) - x_d(t)\right)\right]^T \quad [9]$$

3.2 Model Predictive Control

The utilization of Model Predictive Control (MPC) necessitates the use of a model to anticipate the behavior of the system in the future. The aforementioned model enables the system to estimate the forthcoming actions of the process. Fig 2 illustrates the block diagram of the control system, which is predicated on model-based prediction.

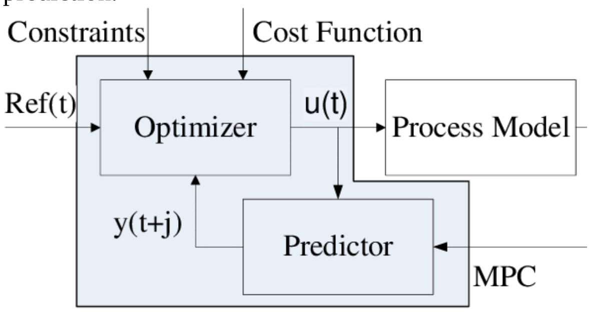

Fig. 2: General MPC block diagram

Control oriented model is a time-invariant linear system therefore discrete time system equation will be as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu_c(k) \\ y(k) &= Cx(k) + Du_c(k) \end{aligned} \qquad [10]$$

According to equation (10), as well as the relationship between the state and control variables $x(k), u_c(k)$, the prediction of the state $x(k+1)$ can be obtained as equation (11).

$$\begin{aligned} &x_{n\times1}(k+1|k) = A_{n\times n}x_{n\times1}(k|k) + B_{n\times m}u_{c_{m\times1}}(k|k) \\ &x(k+2|k) = A^2x(k|k) + ABu_c(k|k) + \\ &Bu_c(k+1|k) \qquad [11] \\ &\vdots \\ &x(k+N_p|k) = A^{N_p}x(k|k) + A^{N_p-1}.B.u_c(k|k) + \cdots \\ &\qquad + A^{N_p-N_c}.B.u_c(k+N_c-1|k) \end{aligned}$$

In the above relationship N_p is the horizon of prediction and N_c is the horizon of control. The prediction equation for a system model can be written as equation (12)

$$X_{nN_p\times1} = F_{nN_p\times n}x_{n\times1}(k|k) + \Phi_{nN_p\times mN_c}U_{mN_c\times1} \quad [12]$$

Where $X \in R^{nN_p}$, $F \in R^{nN_p\times n}$, $\Phi \in R^{nN_p\times mN_c}$ and $U \in R^{mN_c}$ and F and Φ are shown in equations (13),(14)

$$F \triangleq \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{N_p-1} \\ A^{N_p} \end{bmatrix}_{nN_p\times n} \qquad [13]$$

$$\Phi \triangleq \begin{bmatrix} B & \bar{0} & \cdots & \bar{0} \\ AB & B & \cdots & \vdots \\ \vdots & \vdots & \ddots & \bar{0} \\ A^{N_p-1}.B & A^{N_p-2}.B & \cdots & A^{N_p-N_c}.B \end{bmatrix}_{nN_p\times mN_c} \qquad [14]$$

4. Implementation and Results

In order to compare the efficacy of the designed controllers, we employed identical optimization conditions and conducted a comparative analysis of the results. Specifically, we designed the controller by considering the weight matrices Q and R under similar conditions, and defined them as follows:

$Q = diag([10,10,10,10,10])$

$R = 0.1$

The Model Predictive Control (MPC) was designed with a sampling time of one second, a prediction horizon of 30, and a control horizon of 8. However, due to constraints inherent in the aircraft and actuator during cruise flight, we defined the control constraint for the elevator angle as follows:

$$-0.5\ rad \le \delta e \le 0.5\ rad$$

To compare the performance of two controllers in a specific flight maneuver, Fig. 3 illustrates that both controllers exhibited similar path-following characteristics.

Moreover, Fig. 4 and Fig. 5 depict the angle of attack of the aircraft under LQR and MPC controllers, respectively, when the aircraft seeks to adjust its height by 200 meters. Notably, the LQR control's angle of attack reaches the stall angle, causing the aircraft to stall. Conversely, the maximum angle of attack observed under MPC control is lower than the stall angle.

Finally, Fig. 6 and Fig. 7 display the perturbed velocity of the aircraft, which indicate that both controllers exhibit almost identical behavior.

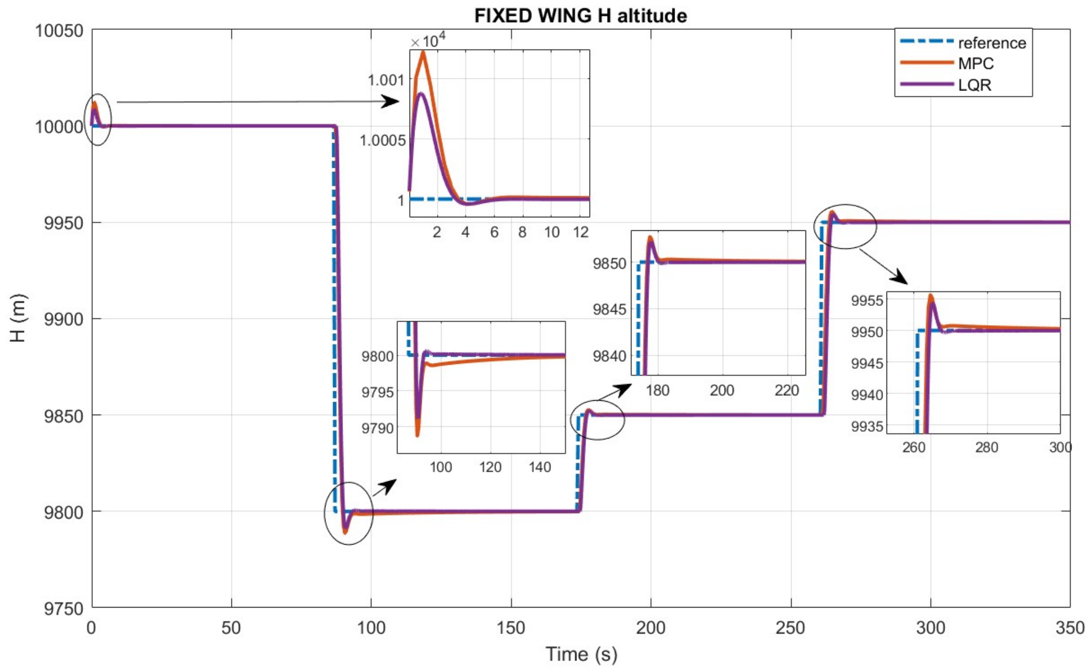

Fig. 3: Aircraft altitude response to the given scenario using the LQR controller and MPC

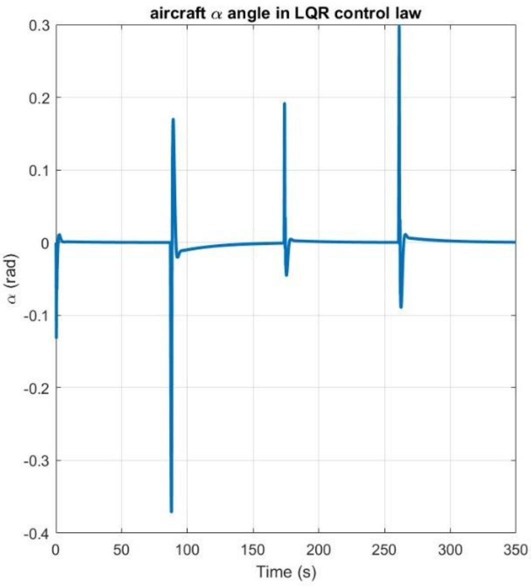

Fig. 4: Aircraft angle of attack in LQR control

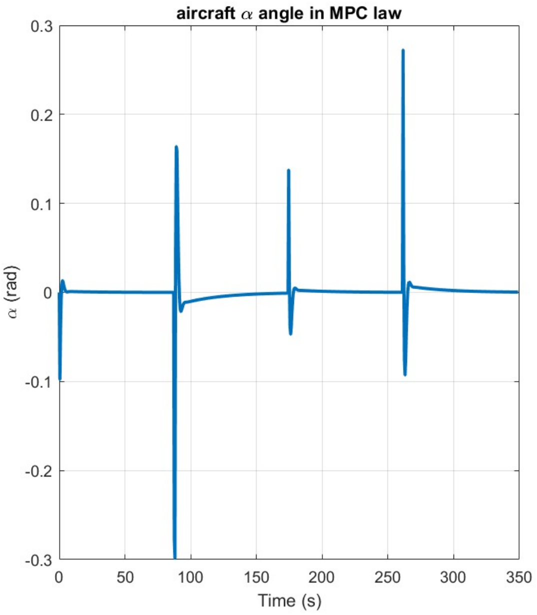

Fig. 5: Aircraft angle of attack in MPC control

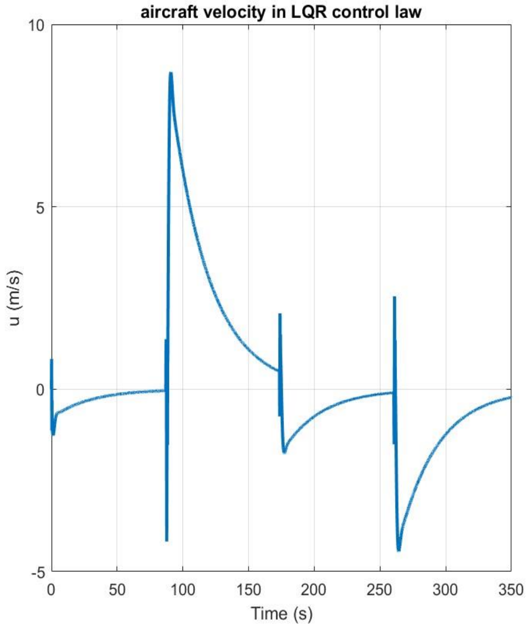

Fig. 6: Perturbed longitudinal speed in LQR control

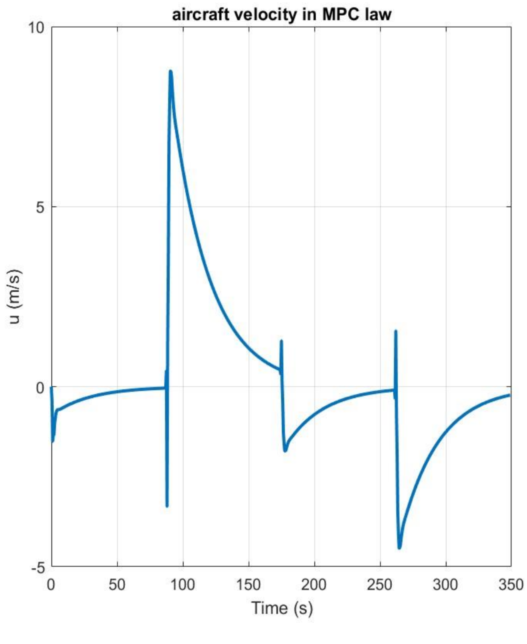

Fig. 7: Perturbed longitudinal speed in MPC control

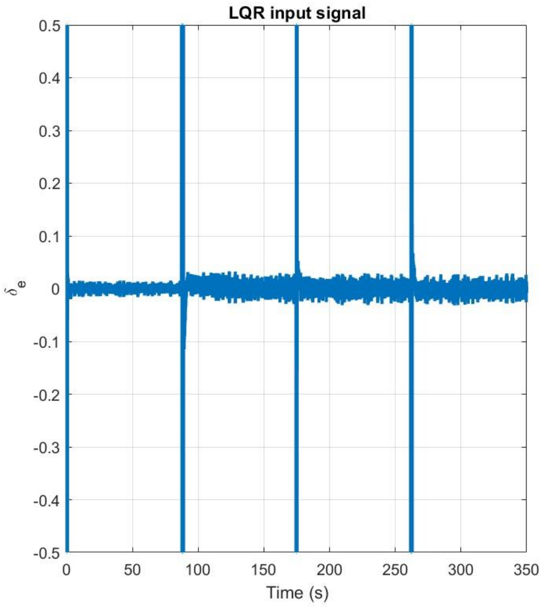

Fig. 8: LQR control signal input (Elevator)

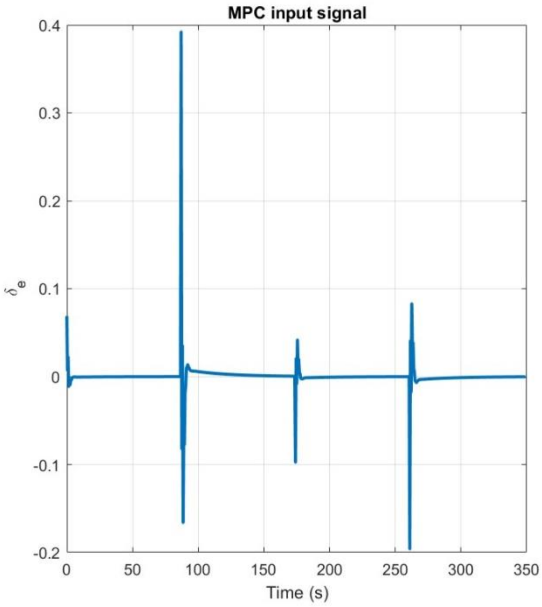

Fig. 9: MPC control signal input (Elevator)

Based on the results depicted in Fig. 8, the Linear Quadratic Regulator (LQR) controller applies the control input (i.e., elevator) over its maximum range. Conversely, the control signal in the Model Predictive Control (MPC), as illustrated in Fig. 9, employs a reduced range to monitor the path. Notably, the control signal in the LQR approach exhibits a noise range of 0.02. This noise could be attributed to operator fatigue or the potential for the operator to disobey the control signal due to the operator's dynamics. However, in an ideal setting, this noise does not exist in the MPC technique.

5. Conclusion

This paper investigates the altitude adjustment of a passenger aircraft using two controllers: Model Predictive Control (MPC) and Linear Quadratic Regulator (LQR), accounting for actuator limitations. After designing the controller for the aircraft's linear dynamics, both controllers exhibit similar altitude adjustment behavior, with a key distinction observed in the angle of attack output. Specifically, the maximum angle of attack under MPC control is 0.27 radians (15°), while under LQR control, it is 0.36 radians (20°). Moreover, MPC control does not result in actuator saturation, whereas LQR control experiences saturation multiple times.

6. References

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