Design a finite-time chattering free attitude controller for rigid spacecraft's without angular velocity measurement using interval type-II fuzzy logic nonsingular terminal sliding mode and nonlinear extended state observer

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Abstract
This paper proposes an intelligent output feedback control method for rigid satellites considering dynamic uncertainties and external disturbances. The dynamics of a rigid satellite are first represented using the modified Rodrigues parameter (MRP) explanation, and then transformed into Lagrangian form in order to build the state-space representation of the dynamics. Because of cost or technical restrictions, angular velocity data are not always accessible for practical application.So angular velocity is considered to be unmeasurable. In order to avoid increasing mathematical calculations and designing separate observers to estimate external disturbances and system states, a non-linear extended state observer has been used to simultaneously estimate disturbances and system states. In order to estimate the system's dynamics in light of the parametric uncertainties, the interval type-2 fuzzy logic approach has been used. The main part of the proposed controller is also composed of the non-singular terminal sliding mode method, which guarantees finite-time stability and elimination of chattering phenomenon. The simulation results of the proposed method have been presented and compared with the results of the methods available in the literature, which shows the efficiency of the method proposed in this paper and the improvement of the results of the methods presented in previous related researches.

Keywords: *spacecraft – attitude control – type-II fuzzy logic – extended state observer – nonsingular terminal sliding mode.*

1. Introduction
The attitude control system of a spacecraft must often meet stringent performance standards in order to be used on space missions. Satellite's attitude control is a critical and practical problem. The fascination stems from its critical role in numerous space operations, including satellite surveillance, space station docking and installation, and spacecraft formation flying [1]. Various techniques for dealing with the problem have been extensively researched during the past few decades including classical linear control, optimal control, model predictive control, nonlinear and intelligent control methods. Sliding mode control (SMC) is a popular option among various techniques for solving aerospace systems control issues. A number of SMC based controllers have been suggested since the initial investigation into their use in spacecraft attitude control was published in [2]. Conventional sliding mode use linear sliding surfaces that only ensure asymptotic system trajectory stability [3]. In contrast, Terminal sliding mode (TSM) employ finite-time stable differential equations to create nonlinear sliding surfaces that converge in finite time and suggested to increase convergence rate and robustness. Nonsingular TSM (NTSM), fast TSM, integral and nonsingular FTSM were all suggested simultaneously as TSM variations [4,5]. Several finite-time attitude controllers have been developed utilizing TSM and its variations [6-10]. When discussing SMC, it's crucial to address the topic of chattering. Reducing the chattering was a primary goal in the incorporation of adaptive estimates into the controller design described in the aforementioned literatures [11]. Incorporating a sign function into the controller, however, rendered the intended control rules in continuous. In addition to adaptive control, the sign function has been replaced with a saturation function in other publications [12]. The key drawback is that the chattering reduction is done at the expense of robustness and tracking precision. Recently, higher order sliding mode (HOSM) control—a powerful strategy for reducing control chattering—has also been used to finite-time attitude control [13-15]. Discontinuous control input is placed on the higher time derivative of the sliding variable in HOSM to dampen the chattering. However, the theoretical analysis and computational load are both substantial for HOSM control. Full information about the sliding mode control method can be read in references [3,4], and also references [5] and [13,14] provide complete information about different types of terminal sliding mode and high-order sliding mode control, respectively. It should be mentioned that the majority of current spacecraft attitude control techniques are dependent on the availability of direct and perfect measurement data of both attitude orientation and angular velocity [1]. However, angular velocity data are not always accessible for practical application due to cost limits or

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implementation constraints. Microsatellites, for example, may be unable to get angular velocity data. As a result of this practical factor, developing partial state feedback attitude control systems for spacecraft is desirable. This problem (the lack of available velocity measurement data) has been discussed in the literature and several strategies have been presented for the construction of angular velocity-free attitude controllers [*].

2. Attitude control challenges and solutions

If we consider the dynamics of spacecraft attitude control in general, similar to many other dynamic systems, we are faced with the following challenges, that an efficient control system must be able to have solutions in its structure to solve the stated challenges, and as much as the control system Designed to solve more of these challenges, it will be more efficient.

A) The presence of dynamic uncertainties including parametric uncertainties such as uncertainty in the values of moments of inertia and non-parametric uncertainties such as external disturbances on the system. To solve this issue, to estimate the system dynamics and disturbances and reduce the effect of the presence of these uncertainties in the performance of the control system in the structure of the control system, neural networks, fuzzy logic, fuzzy- neural networks and various types of disturbance observers can be used.

B) The unavailability of some system states due to the lack of measurement sensors or the failure of the relevant sensors, or the lack of use of sensors due to their high costs, or the unwillingness to measure some system states due to the errors in measuring system states through sensors. To solve this issue, we can use a variety of state observers depending on the type of problem, such as reduced order state observers, extended state observers, state observers based on conventional sliding mode and high order sliding mode and combining it with methods based on neural networks and fuzzy logic.

C) The stability of the control system in terms of the time required to reach the stable state, which includes types of stability such as asymptotic stability, finite time stability and fixed time stability. Finite time and fixed-time stability in controlling the attitude of spacecraft's has an advantage over asymptotic stability due to the rapid and sequential maneuvers that may be performed depending on the type of mission. To solve this issue, control methods with finite time stability can be used, such as methods based on high-order sliding mode, including terminal sliding mode, non-singular terminal sliding mode, and super twisting sliding mode.

D) Among other things that can be considered for the design of an efficient control system for spacecraft's is to consider the saturation and faults of the actuators to consider the ability of the actuators to produce the designed control inputs in the practical implementation of the controller and the design of the fault tolerant controllers, which of course is not one of the challenges we will deal with in this research.

According to the above topics in the present article finite-time chattering free attitude controller designed for rigid spacecraft's without angular velocity measurement. We use interval type-2 fuzzy logic for dynamic modeling of an uncertain spacecraft's system, a nonlinear extended state observer for the estimation of angular velocities and nonsingular terminal sliding mode for finite time and chattering free tracking controller.

3. Attitude kinematics and dynamics description

Various methods have been used to describe satellite dynamics and kinematics in the literature, each of which has advantages and disadvantages. Among the methods used to describe the kinematics and dynamics of satellites, we can mention the Eulerian angels, the Euler-Rodrigues parameters (Quaternion formulation), the Cayley-Klein parameters, and the Cayley-Rodrigues parameters [21, 22]. Singularities are a recognized feature of any parameterization in three dimensions (e.g., the Rodrigues parameters are singular for 180 degree rotations). Several considerations, including the kind of spacecraft rotation maneuver, processing needs, physical representation insight, etc., influence the selection of parameters.

Most controls applications call for a parameterization with the singularity distant from the origin. The quaternion representation is a popular parameterization for attitude assessment. The kinematic equations are linear with regard to angular velocities when utilizing quaternions, and there are also no singularities for any rotation of the Eigen axis and an algebraic attitude matrix. However, the quaternion components are not minimum since the quaternion parameterization uses four components to express the attitude motion (dependent). This results in the need that the quaternion have a unit norm [21, 22].

Avoiding singularities in the kinematic equations is a benefit of the quaternion formulation. However, non-minimal parameterization results from the usage of quaternions because of the need for an additional parameter. In particular, spacecraft control applications have lately made use of the modified Rodrigues parameters (MRP), which allow for rotations of up to 360 degrees [23-25]. The minimum (i.e., three-dimensional) parameterization is given by the Rodrigues parameters. This parameterization, however, is hampered for rotations of very large angles due to the existence of a singularity for rotations of 180 degrees. Applying sequential rotations of fewer than 180 degrees every revolution may help to solve this problem. However, the entire strategy can call for more control authority and power than is required. Using this attitude representation has the following advantages: 1) rotations of up to 360 degrees are feasible; and 2) the parameters constitute a minimum parameterization [21]. The increase in power consumption can be reduced by using intelligent control methods as well as chattering-free control methods. Also, in many cases, there is a need to design observers and estimators to estimate the attitude of satellites, and the use of modified Rodrigues parameters method compared to the quaternion method will reduce the volume of calculations and simplify the control design process in terms of mathematical calculations.

Therefore, in the current research, we will use this modeling method to describe the dynamics and kinematics of the satellite according to the type of controller considered.

If we assume that the $\sigma \in \mathcal{R}^3$ represents the MRP, which is defined as the following relationship [10, 26]:

$$\sigma = [\sigma_1 \quad \sigma_2 \quad \sigma_3]^T = \hat{e} \tan\frac{\theta}{4} \tag{1}$$

Where $\hat{e} = [e_1 \quad e_2 \quad e_3]$ denotes a rotation of the Euler axis in a body frame and θ equals to rotation about the main axis. The system's attitude kinematics may be characterized in terms of σ as follows:

$$\dot{\sigma} = \Gamma(\sigma)\omega \tag{2}$$

Where ω is the angular velocity components represented in a body axis frame $[x \quad y \quad z]$ relative to an inertial frame $[X \quad Y \quad Z]$ and $\Gamma(\sigma)$ can be written as follows [*]:

$$\Gamma(\sigma) = \frac{1}{4}[(1-\sigma^T\sigma)I_{3\times3} + 2S^*(\sigma) + 2\sigma\sigma^T] \tag{3}$$

$I_{3\times3}$ is a 3×3 identity matrix, whereas S^* is a skew-symmetric matrix expressed as follows:

$$S^*(\sigma) = \begin{bmatrix} 0 & -\sigma_3 & \sigma_2 \\ \sigma_3 & 0 & -\sigma_1 \\ -\sigma_2 & \sigma_1 & 0 \end{bmatrix} \tag{4}$$

Considering the control inputs $u(t) \in \mathcal{R}^3$ and the external disturbances $d(t) \in \mathcal{R}^3$,the rigid spacecraft's dynamics can be expressed as follows [10, 26]:

$$J\dot{\omega} = -S^*(\omega)J\omega + u(t) + d(t) \tag{5}$$

Where $J \in \mathcal{R}^3$ is the matrix of the moment of inertia and $S^*(\omega)$ is the skew-symmetric matrix of the angular velocity. Using Eqs. (2) and (5), the following is a Lagrangian form of the dynamics of the spacecraft's attitude stabilization[8,10,26]:

$$M(\sigma)\ddot{\sigma} + C(\sigma,\dot{\sigma})\dot{\sigma} = \tau + \tau_{ext} \tag{6}$$

The various terms in the above equation can be expressed as follows:

$$M(\sigma) = \Gamma(\sigma)^{-T}J\Gamma(\sigma)^{-1} \tag{7}$$

$$\begin{aligned} C(\sigma,\dot{\sigma}) = \; & -\Gamma(\sigma)^{-T}J\Gamma(\sigma)^{-1}\dot{\Gamma}(\sigma)\Gamma(\sigma)^{-1} \\ & -\Gamma(\sigma)^{-T}S^*(J\omega)\Gamma(\sigma)^{-1} \end{aligned} \tag{8}$$

$$\tau = \Gamma(\eta)^{-T}u(t) \tag{9}$$

$$\tau_{ext} = \Gamma(\eta)^{-T}d(t) \tag{10}$$

The general form of the satellite dynamics expressed in Eq. (6) is similar to the general form of the equations related to the dynamics of robotic systems, which is obtained through the Euler-Lagrange method. The dynamic equations of robotic systems that are written in this form have some features that are also true for the satellite dynamic equations. Positive definite, symmetric, and boundedness of the inertial matrix (J), skew-symmetry of the $\dot{M}(\sigma) - 2C(\sigma,\dot{\sigma})$ matrix and boundedness of the matrix $C(\sigma,\dot{\sigma})\dot{\sigma}$ can be mentioned among these properties.

In controller design for nonlinear systems, the state space form of dynamic equations is usually used. The state space form of the satellite dynamic equations can be expressed as follows with the assumption of $x_1 = \sigma$ and $x_2 = \dot{\sigma}$:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + g(x)u + d^* \end{cases} \tag{11}$$

Where $f(x) = M^{-1}(\sigma)C(\sigma,\dot{\sigma})\dot{\sigma} \in \mathcal{R}^3$, $g(x) = M^{-1}(\sigma)\Gamma(\sigma)^{-T} \in \mathcal{R}^3$and $d^* = M^{-1}(\sigma)\Gamma(\sigma)^{-T}d(t) \in \mathcal{R}^3$.

4. Spacecraft Error Dynamics

Following, the relative attitude error dynamics are developed to model the tracking process of the proposed control technique. Consider $\sigma_d \in \mathcal{R}^3$ to be the satellite's desired attitude in a body-fixed frame. The term for attitude error $\sigma_e \in \mathcal{R}^3$ is [8, 26]:

$$\sigma_e = \frac{(1-\sigma_d^T\sigma_d)\sigma - (1-\sigma^T\sigma)\sigma_d + 2S'(\sigma)\sigma_d}{1 + \sigma^T\sigma\sigma_d^T\sigma_d + 2\sigma_d^T\sigma} \tag{12}$$

Consider $\omega_d \in \mathcal{R}^3$ to be the desired angular velocity in the fixed frame of the body. $\omega_e \in \mathcal{R}^3$ is the equation for the angular velocity error:

$$\omega_e = \omega - Z(\sigma_e)\omega_d \tag{13}$$

$Z(\sigma_e)$ in the above equation can be written as follows:

$$Z(\sigma_e) = Z(\sigma)Z(\sigma_d) \tag{14}$$

In which expressions $Z(\sigma)$ and $Z(\sigma_d)$ are written as[8]:

$$\begin{aligned} Z(\sigma) = I_{3\times3} - 4\frac{1-\sigma^T\sigma}{(1+\sigma^T\sigma)^2}S^*(\sigma) \\ +8\frac{S^*(\sigma)}{1+\sigma^T\sigma}S^*(\sigma) \end{aligned} \tag{15}$$

$$\begin{aligned} Z(\sigma_d) = I_{3\times3} - 4\frac{1-\sigma_d^T\sigma_d}{(1+\sigma_d^T\sigma_d)^2}S^*(\sigma_d) \\ +8\frac{S^*(\sigma_d)}{1+\sigma_d^T\sigma_d}S^*(\sigma_d) \end{aligned} \tag{16}$$

The satellite system's attitude kinematics, given an attitude error σ_e and an angular velocity error ω_e, may be expressed in a form analogous to Eq. (2):

$$\dot{\sigma}_e = G(\sigma_e)\omega_e \tag{17}$$

The satellite system's attitude dynamics using error coordinates may be expressed as Eq. (5) [8]:

$$\begin{aligned} J\dot{\omega}_e = & -S^*(\omega_e + Z(\sigma_e)\omega_d)J(\omega_e + Z(\sigma_e)\omega_d) \\ & -J\dot{Z}(\sigma_e)\omega_d - JZ(\sigma_e)\dot{\omega}_d + u(t) + d(t) \end{aligned} \tag{18}$$

The Lagrangian form of the above equation can be written as follows [8, 10]:

$$M(\sigma_e)\ddot{\sigma}_e + C(\sigma_e,\dot{\sigma}_e)\dot{\sigma}_e + G(\sigma_e) = \tau + \tau_{ext} \tag{19}$$

The various terms in the above equation can be expressed as follows [8]:

$$M(\sigma_e) = \Gamma(\sigma_e)^{-T}J\Gamma(\sigma_e)^{-1} \tag{20}$$

$$C(\sigma_e,\dot{\sigma}_e) = \Gamma(\sigma_e)^{-T}J\dot{\Gamma}(\sigma_e)^{-1} -\Gamma(\sigma_e)^{-T}S^*(J\omega_e)\Gamma(\sigma_e)^{-1} -\Gamma(\sigma_e)^{-T}\big(S^*(JZ(\sigma_e)\omega_d)\big)\Gamma(\sigma_e)^{-1} +\Gamma(\sigma_e)^{-T}(JS^*(Z(\sigma_e)\omega_d) + S^*(Z(\sigma_e)\omega_d)J)\Gamma(\sigma_e)^{-1} \quad (21)$$

$$G(\sigma_e) = \Gamma(\sigma_e)^{-T}(S^*(Z(\sigma_e)\omega_d)JZ(\sigma_e)\omega_d) +JZ(\sigma_e)\dot{\omega}_d \quad (22)$$

$$\tilde{\tau} = \Gamma(\sigma_e)^{-T}u(t) \quad (23)$$

$$\tilde{\tau}_{ext} = \Gamma(\sigma_e)^{-T}d(t) \quad (24)$$

Now, similar to the previous section, we need to obtain the form of the state space of the spacecraft error dynamics. By defining state variables as $x_1 = \sigma_e$ and $x_2 = \dot{\sigma}_e$ the state space form of the above equations can be written as Eqs. (11), where $f(x) = M(\sigma_e)^{-1}C(\sigma_e,\dot{\sigma}_e)\dot{\sigma}_e \in \mathcal{R}^3$, $g(x) = M(\sigma_e)^{-1}\Gamma(\sigma_e)^{-T} \in \mathcal{R}^3$ and $d^* = (M(\sigma_e)^{-1}\Gamma(\sigma_e)^{-T}d(t) - M(\sigma_e)^{-1}\Gamma) \in \mathcal{R}^3$.

Therefore, the final form of the equation that is based on the spacecraft error dynamics and is used to design the proposed controller can be expressed as follows assuming $x = [x_1, x_2] = [\sigma_e, \dot{\sigma}_e]$:

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = f(x) + g(x)u + d\\ y = x\end{cases} \quad or \quad \begin{cases}\ddot{x} = f(x) + g(x)u + d\\ y = x\end{cases} \quad (25)$$

Whereas d is bounded unknown external disturbance $|d| \le \delta_d$. Considered here are the two functions denoted by the $f(x)$ and $g(x)$, each of which may be rewritten as the sum of a nominal function and an undetermined but limited uncertainty:

$$\begin{cases}f(x) = f_0(x) + \Delta f(x); |\Delta f(x)| \le \delta_f\\ g(x) = g_0(x) + \Delta g(x); |\Delta g(x)| \le \delta_g\end{cases} \quad (26)$$

By substituting Eq.(26) in Eq.(25), it can be written:

$$\begin{cases}\ddot{x} = f_0(x) + g_0(x)u + D\\ y = x\end{cases} \quad (27)$$

Where $D = \Delta f(x) + \Delta g(x)u + d$ denote the total disturbances of the system. By assuming an upper bound for control inputs such as $|u| \le \delta_u$ it can be written:

$$|D| \le |\Delta f + \Delta g\delta_u + \delta_d| \le \left|\delta_f + \delta_g\delta_u + \delta_d\right| \le \Delta_D \quad (28)$$

5. Interval Type-2 Fuzzy Logic (IT2FL):

There are two main categories of uncertainty in satellite attitude control: parametric, like the uncertainty in the inertia matrix, and non-parametric, such external disturbances like air drag, solar pressure, and gravitational perturbations. Here, it's essential to reduce the impact of these uncertainties on the effectiveness of the satellite attitude controller [7].

The idea of fuzzy logic, which has been widely employed to reduce the consequences of uncertainties in control issues by giving a reasonable assessment of the ambiguities existing in the situation, is one solution for the broader problem of uncertainties. You may find illustrations of this inference in [27]. Different kinds of fuzzy sets might be used for this purpose. The type-1 fuzzy sets (T1FS) and the corresponding FLC perform less than optimally because the type-1 fuzzy sets (T1FS) and the corresponding FLC cannot solely handle uncertainties present in dynamic environments such as the problem at hand because these uncertainties have multiple sources affecting various components of the control system [28-30].

On the other hand, T2FL extends the range of fuzziness from the input data to the membership functions, generalizing the traditional T1FL. In contrast to a type-1 fuzzy set (T1FS), where the membership grade is a crisp integer in [0,1], a type-2 fuzzy set (T2FS) has a fuzzy membership function, or membership value, for each element. It is the new third dimension of T2FS and the footprint of uncertainty that give extra degrees of freedom that make it feasible to directly describe and manage uncertainties [31-32]. The membership functions of interval T2FS are three dimensional and contain a footprint of uncertainty. You may find further comparisons between type-1 and type-2 FLCs in [7].

Since IT2FLC was created to deal with extremely uncertain dynamics in a more effective manner than T1FLCs [7], it may be said to be superior than T1FLC due to the nature of real-world issue uncertainties and the fact that they are often unable to be estimated adequately [7]. A T2FLC might also be a preferable option for the system, taking into account the ability of this type to manage uncertainties, given the significant degree of uncertainty in the dynamics of the model and the data transmittance channels inherent in the current case. It has been shown in several earlier studies that the efficiency of the IT2FLC in managing them decreases as the degree of uncertainty and imprecision rises. Additionally, it has been shown in several applications that T2FLCs outperformed T1FLCs [32-35]. IT2FLCs provide smoother steady-state control signals and superior measurement noise performance compared to T1FLCs. However, this greater performance is offset by the iterative Karnik-Mendel (KM) algorithms' high computing cost, a problem that has, of course, been addressed in several studies and the development of numerous computational techniques to solve [36-38].

We will utilize the IT2FL to estimate the uncertain system dynamics, taking into account the factors indicated above, the nature of the issue under investigation in the current research, and its operating context.

For fuzzy dynamic modeling if we consider an n-order dynamic system like Eq. (27) as follow:

$$\begin{cases} x^{(n)} = f_0(x) + g_0(x)u + D \\ y = x \end{cases} \quad (29)$$

A collection of fuzzy If-Then rules describes a TS type fuzzy model of a dynamic system. One distinctive aspect of these systems is that they explain each conclusion in terms of a linear system representing the dynamics of the system under consideration at that particular time. The i^{th} rule of the fuzzy model-type-1 in its original form as presented by Takagi and Sugeno is worded as follows [39]:

$$If\ x_1(t)\ is\ F_i^1\ and\ \dots\ and\, x_n(t)\ is\, F_i^n\ Then$$

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad (30)$$

In this expression, F_i^j represents the j^{th} fuzzy set, and indicates the quantity of rules. Regulations for building are based mostly on experience and judgment from humans. However, since each person's experience and interpretation of the event is unique, we are sometimes forced to rely on inaccurate information and generate rules with questionable premises or conclusions.

There will be confusion about the regulations as a result of this. By integrating the idea of fuzzy-type-2 to the dynamics of the model description in Eq. (29), we may take greater use of the advantages of fuzzy logic and better account for these uncertainties. To be more specific, the i^{th} rule of the fuzzy type-2 model is expressed as follows [39]:

$$If\ x_1(t)\ is\ \tilde{F}_i^1\ and\ \dots\ and\, x_n(t)\ is\, \tilde{F}_i^n\ Then$$

$$\begin{cases} \dot{x}(t) = \tilde{A}_i x(t) + \tilde{B}_i u(t) \\ y(t) = \tilde{C}_i x(t) \end{cases} \quad (31)$$

where $\tilde{F}_i^1$ is the i^{th} type-2 fuzzy interval, i denotes the number of rules, $x(t) = [x_1(t), \dots, x_n(t)]^T$ denotes the state vector, $u(t)$ denotes the control inputs vector, $y(t)$ denotes the system output vector, A_i denotes the state matrix, B_i denotes the system input matrix, and C_i denotes the system output matrix. The fuzzy model will show up as a weighted average of local linear models for pair-state drive $(x(t), u(t))$ data [39]:

$$\begin{cases} \dot{x}(t) = \dfrac{\sum_{i=1}^{R} w_i(x(t))\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{R} w_i(x(t))} \\ y(t) = \dfrac{\sum_{i=1}^{R} w_i(x(t)) C_i x(t)}{\sum_{i=1}^{r} w_i(x(t))} \end{cases} \quad (32)$$

Where $w_i(\underline{x}(t)) = \prod_{j=1}^{n} \tilde{\mu}_i^j\big(x_j(t)\big)$; for $i = 1,2, \dots, R$ and $\tilde{\mu}_i^j\big(x_j(t)\big)$ identifies the fuzzy set's membership function $\tilde{F}_i^1$. It can be said for $\forall t \geq 0$ [39]:

$$\begin{cases} w_i(x(t)) = \big[\underline{w}_i(x(t)) \overline{w}_i(x(t))\big] \\ \underline{w}_i(x(t)) = \underline{\mu}_1^j(x_1) \times \dots \times \underline{\mu}_n^j(x_n)\big] \\ \overline{w}_i(x(t)) = \bar{\mu}_1^j(x_1) \times \dots \times \bar{\mu}_n^j(x_n) \end{cases} \quad (33)$$

Using the following auxiliary relation:

$$\psi_i(x(t)) = \frac{1}{2}\left(\frac{\underline{w}_i(x(t))}{\sum_{i=1}^{R} w_i(x(t))} + \frac{\overline{w}_i(x(t))}{\sum_{i=1}^{R} \overline{w}_i(x(t))}\right) \quad (34)$$

It is possible to rewrite the TS fuzzy model, which is described in Eq. (31), as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{R} \psi_i(x(t))\{A_i x(t) + B_i u(t)\} \\ y(t) = \sum_{i=1}^{R} \psi_i(x(t)) C_i x(t) \end{cases} \quad (35)$$

As a result, Takagi and Sugeno suggested a fuzzy dynamic model comprising rules that ended in the form of a linear state representation system to regulate the calculation of the nominal model. Type-2 fuzzy logic was included into the same model to get this result.

Considering Eq. (28) from the i^{th} rule, we may deduce the type-2 nominal design, which is represented as:

$$If\ x\ is\ \psi_1^i\ and\ \dot{x}\ is\ \psi_2^i\ and\ \dots x^{(n-1)}\ is\ \psi_n^i\ Then$$

$$x^{(n)} = A_i(n, 1{:}\, x)x + B_i(n)u \quad (36)$$

Where $\psi_j^i (j = 1,2, \dots, n)$ is the j^{th} type-2 fuzzy set interval of the i^{th} rule and $A(n, 1{:}\, n) = A(n, j)_{1 \le j \le n}$ is the n^{th}row of the matrix A.

For a given control state (x, u), the fuzzy model of the resultant system may be interpreted as a weighted average of the many local models [40,41]. If the product is used as an inference engine, the whole center of the technique for the type of reduction, and the center of gravity defuzzification, the output of the system is provided by the following equations:

$$x^{(n)} = \frac{\sum_{i=1}^{R} w^i (A_i(n, 1{:}\, n)x)}{\sum_{i=1}^{R} w^i} + \frac{\sum_{i=1}^{R} w^i (B_i(n)u)}{\sum_{i=1}^{R} w^i} \quad (37)$$

Where $f_0(x) = \frac{\sum_{i=1}^{R} w^i (A_i(n,1:n)x)}{\sum_{i=1}^{R} w^i}$ and $g_0(x) = \frac{\sum_{i=1}^{R} w^i (B_i(n)u)}{\sum_{i=1}^{R} w^i}$ and within the context of the fuzzy nominal model, the variable w^i refers to the activation interval.

6. Nonlinear Extended State Observer (N-ESO)

A nonlinear extended state observer is presented to circumvent the issue that not all system states are available for measurement in spacecraft's attitude control processes. In this scenario, the system's state may be determined by linear ESO as follow [42]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + g_0(x)u(t) + f_0(t) \\ \dot{x}_3 = \dot{D}(t) \end{cases} \quad (38)$$

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - \rho_1(\hat{x}_1 - x_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 - \rho_2(\hat{x}_1 - x_1) + g_0(x)u + f_0(x) \\ \dot{\hat{x}}_3 = -\rho_3(\hat{x}_1 - x_1) \end{cases} \quad (39)$$

Where $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ are the estimated states, ρ_1 , ρ_2 and ρ_3 are positive constants, and they have the effect of turning the real sections of the eigenvalues of the subsequent polynomial into a negative value [42].

$$s^3 + \rho_1 s^2 + \rho_2 s + \rho_3 = 0 \tag{40}$$

In order to improve the estimation performance of the linear ESO here we use a nonlinear function called $fal(.)$ based on equation (38), and it constructs a nonlinear ESO in the following way [43, 44]:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - \rho_1 fal(\tilde{x}_1, \lambda_1, \varepsilon_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 - \rho_2 fal(\tilde{x}_1, \lambda_2, \varepsilon_2) + g_0(x)u + f_0(x) \\ \dot{\hat{x}}_3 = -\rho_3 fal(\tilde{x}_1, \lambda_3, \varepsilon_3) \end{cases} \tag{40}$$

Where $\tilde{x}_1 = \hat{x}_1 - x_1$, $0 \leqslant \lambda_1, \lambda_2, \lambda_2 \leqslant 1$ and $\varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$. The $fal(.)$ Function is defined as:

$$fal(x, \lambda, \varepsilon) = \begin{cases} x\varepsilon^{1-\lambda}, & |x| \leqslant \varepsilon \\ |x|^{\lambda} sgn(x) & |x| \geqslant \varepsilon \end{cases} \tag{40}$$

λ is a nonlinear factor in this formula, and ε is the filter factor. For the convergence analysis of the designed N-ESO readers referred to [44, 45].

7. Nonsingular Terminal Sliding Mode Control

For the design of the NTSM controller, we first define the sliding surface as follows, taking into account Eq. (25) [46]:

$$S = x_1 + \frac{1}{\beta} \boldsymbol{x}_2^{\frac{p}{q}} \tag{41}$$

Where $\beta > 0$ *and* $p > q, 1 < \frac{p}{q} < 2$ are positive odd numbers. Similar to the TSM the following is a sufficient need for NTSM to exist [46]:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t} s^2 < -\eta |s| \tag{42}$$

Where $\eta > 0$ and the control input function designed as follow [46]:

$$u = -g_0{}^{-1}(\boldsymbol{x}) \left(f_0(\boldsymbol{x}) + \beta \frac{q}{p} x_2^{2-p/q} + (\Delta_d + \eta)\mathrm{sgn}(s) \right) \tag{43}$$

In [12], various functions have been proposed to replace the sign function in order to eliminate chattering, among which we use function $\frac{s}{\sqrt{s^2+1}}$ to replace the sign function.

The sliding mode $s = 0$ will be reached by the system states in the finite time t_r if $s(0) \neq 0$, which meets the following conditions [46]:

$$t_r \leqslant \frac{|s(0)|}{\eta} \tag{44}$$

When $s = 0$ in TSM is achieved the finite time t_s required to go from point $x_1(t_r) \neq 0$ to point $x_1(t_s + t_r) = 0$ is expressed as [46]:

$$t_s = -\beta^{-1} \int_{x_1(t_r)}^{0} \frac{\mathrm{d}x_1}{x_1^{q/p}} = \frac{p}{\beta(p-q)} |x_1(t_r)|^{1-q/p} \tag{45}$$

The finite time required to achieve the equilibrium point $x_1 = 0$ in the NTSM is the same as in Eq. (45) after the switching line is reached. The NTSM manifold may thus be achieved in a limited amount of time and in a finite time, the states in the NTSMC will approach zero. The conclusion is that the sliding mode $s = 0$ may be achieved in finite time from anyplace in the phase plane. So the considered NTSM is globally finite time stable.

Readers referred to [46] for further study. In order to prove the stability of the control system designed in this paper, according to the use of the N-ESO to estimate the system states and the disturbances on it, it is enough to place the values estimated by the observer instead of the variables in the process of proving the stability of NTSM method which is fully stated in references [46] and we refrain from bringing it here in order to avoid making the material bulky.

8. Simulation and Results:

The simulation is provided in this part to demonstrate the efficacy and applicability of the suggested controller. Considering that the results of the proposed method will be compared with the results of reference [10] the spacecraft's desired attitude command and other simulation parameters is selected as [10]:

$$\sigma_d = 0.5[\sin(0.01t), -\cos(0.01t), \sin(0.01t)]^T$$

$$\sigma(0) = [0.7 \quad 0.5 \quad -0.3]^T$$

$$\omega(0) = [-0.001 \quad 0.001 \quad -0.001]^T \mathrm{rad/s}$$

$$d(t) = 10^{-3} \begin{bmatrix} \sin(0.01t) \\ 0.3\cos(0.02t) \\ 0.5\sin(0.02t) \end{bmatrix} \; and \; J = \begin{bmatrix} 3.06 & 1 & 0.4 \\ 1 & 3 & 1 \\ 0.4 & 1 & 3.95 \end{bmatrix}$$

The simulation results of the proposed control method and its comparison with the results of [10] are shown in Figures 1 to 6. As it is clear in the figures, the tracking is done well and the control method proposed in this article has a better tracking and control input performance than the method proposed in reference [10].

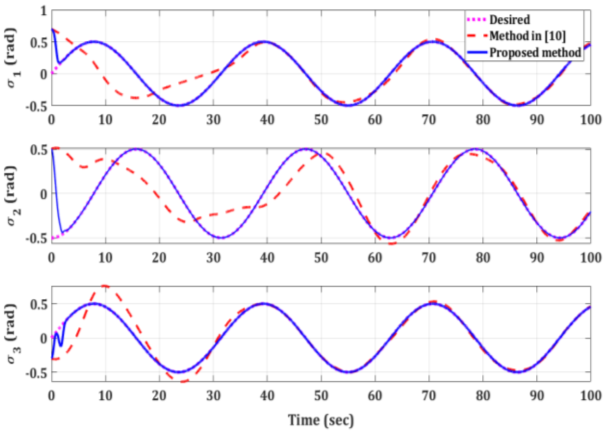

Fig. 1: Attitude tracking performance

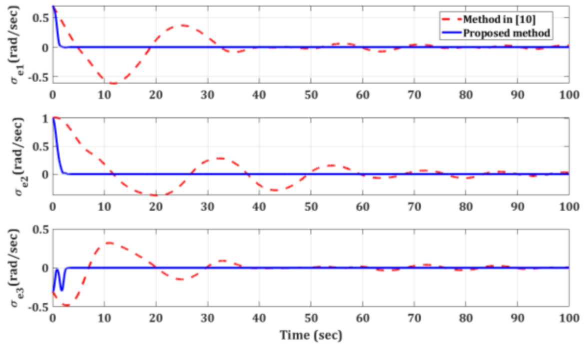

Fig. 2: Attitude tracking errors

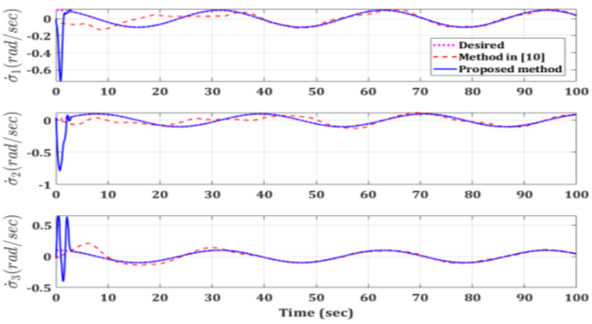

Fig. 3: Angular velocity tracking performance

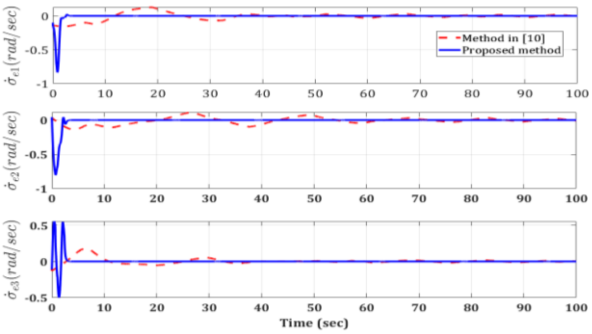

Fig. 4: Angular velocity tracking errors

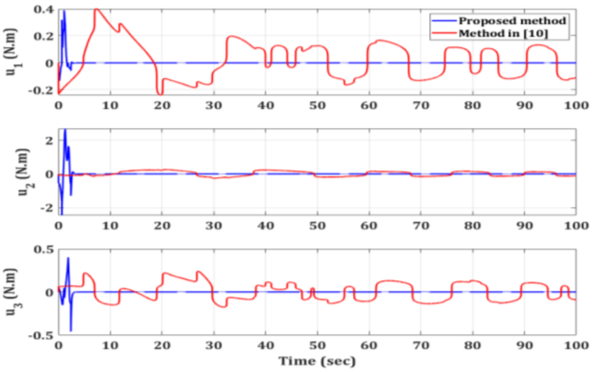

Fig. 5: Control inputs

9. Conclusions

A finite-time angular velocity measurement free attitude control method without chattering proposed in this paper. We used NSTM in combination whit N-ESO and IT2FL for constructing an intelligent attitude controller. The simulation results show the effectiveness of the proposed control method in spacecraft control.

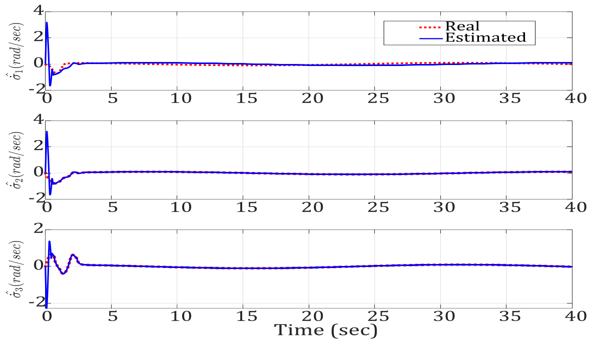

Fig. 6: N-ESO performance

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