A correction on the Lambert targeting problem in the perturbed space environment

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Abstract

A novel correction algorithm is presented in this article to modify the Lambert targeting problem (LTP). To take into consideration space perturbations, the proposed LTPC (LTP Correction) algorithm combines the LTP with the shooting method and Particle Swarm Optimization (PSO). Once the LTP is applied, the five consecutive iterative stages are established to modify it. The novel developed algorithm is called LTPC, which modifies LTP to regard space perturbations.

The Earth's oblateness is considered as a perturbation of interest. This is while the proposed method can regard any space perturbations. The results designate that the delta true anomaly between the desired arrival true anomaly and the arrival true anomaly of the proposed LTPC is much less than the delta true anomaly between the desired arrival true anomaly and the arrival true anomaly of the classical LTP. This is while there is no significant difference between the flight time and total requirement velocity change between LTP and the proposed LTPC. Therefore, the proposed LTPC has a much higher targeting accuracy, which is due to the ability to consider space perturbations

Keywords: *Lambert targeting problem correction- Shooting method- PSO optimization- Earth oblateness.*

1. Introduction

Lambert Targeting Problem (LTP) problem is the classical two-point boundary value problem (TPBVP) in celestial mechanics, that was first posed and solved by Johann Heinrich Lambert in 1761 [1]. It is known to have a unique solution for the fractional orbit transfer between prescribed positions in a prescribed "time of flight." Solving the problem requires determining the orbital arc (typically, solving for the initial velocity) connecting a prescribed initial position and final position, which correspond to the specified flight time. Many investigations established the LTP. A uniform solution to the Lambert problem is investigated by Kriz [2]. Lambert problem solution in the hill model of motion by Sukhanov and Prado [3]. They explain the LTP in the Lambert problem in the restricted three-body problem. Leeghim and Jaroux studied the Energy-optimal solution to the lambert problem [4]. New solutions for the lambert problem using regularization and Picard iteration are done by Woollands and Younes [5]. Thompson and Rostowfske express practical constraints for the applied Lambert problem [6]. Constrained multiple-revolution Lambert's problem is investigated by Zhang and Moratari. A fixed-time, multiple-revolution Lambert's problem is solved under given constraints in their study. A solution based on a dynamical approach for the multiple-revolution lambert problem is investigated by Arlulkar and Naik [7]. Li and Han explained multiple-revolution solutions of the transverse-eccentricity-based lambert problem [8]. Uncertain lambert problem with a probabilistic approach is explained by Adurthi and Majji [9]. All the mentioned research established the LTB ignoring the effects of space perturbations. But it should be noted that the effect of perturbations, however small, can be the cause of success or failure of an space mission. Therefore, A novel correction algorithm is presented in this article to modify Lambert targeting problem (LTP). To take into consideration space perturbations, the proposed LTPC (LTP Correction) algorithm combines the LTP with the shooting method [10], [11] and Particle Swarm Optimization (PSO) [12], [13]. Once the LTP is applied, the five consecutive iterative stages are established to modify it. The novel developed algorithm is called LTPC, which modifies LTP to regard space perturbations. The Earth's oblateness is considered as a perturbation of interest [14]–[16]. This is while the proposed method can regard any space perturbations. The results indicate that the proposed LTPC has a much higher targeting accuracy, which is due to the ability to consider space perturbations

2. Lambert targeting problem (LTP)

Briefly, the Lambert Targeting problem (LTP) determines the transfer orbit between the initial orbit at the departure point $p_1(\vec{r}_d, \vec{v}_d)$ and final orbit at the arrival point $p_2(\vec{r}_a, \vec{v}_a)$ by having transfer time $t_{lambert}$

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[4], [17], [18]. In fact, LTP finds the desired velocity vector $(\vec{V}_{LTP_d}, \vec{v}_{LTP_r})$ of transfer orbit at $\vec{r}_d$, $\vec{r}_a$. The orbital elements of transfer orbit are also obtained from the state vector at $(\vec{r}_a, \vec{V}_{LTP_a})$ or $(\vec{r}_a, \vec{V}_{LTP_d})$[19]. The required total velocity changes for the deployment of a satellite equals:

$$\Delta\vec{V}_{Sat1} = \left|\vec{V}_{LTP_a} - \vec{v}_a\right| + \left|\vec{V}_{LTP_d} - \vec{v}_d\right| \qquad (1)$$

One of the most critical requirements of designing a mission is trying to make it operational. LTP cannot be considered as a practical scheme in the mission design due to the following disadvantages:

- LTP cannot consider space perturbations due to the use of Kepler's equations of motion.
- Due to the accumulative trait of the perturbations, LTP is not suitable for long-term missions.

Therefore, since accurate targeting is vital in the orbital transferring and rendezvous maneuvers, the LTP is not appropriate for practical space missions.

Suppose that the Earth's oblateness is chosen as the space perturbations of interest in this research. The perturbed equations of motion due to the oblate perturbations are as follows [20]:

$$\begin{aligned}
\ddot{x} &= -\frac{\mu}{r^3}x + \frac{3J_2\mu R^2}{r^4}\left[\frac{x}{r}\left(\frac{5z^2}{r^2} - 1\right)\right] \\
\ddot{y} &= -\frac{\mu}{r^3}y + \frac{3J_2\mu R^2}{r^4}\left[\frac{y}{r}\left(\frac{5z^2}{r^2} - 1\right)\right] \qquad (2) \\
\ddot{z} &= -\frac{\mu}{r^3}z + \frac{3J_2\mu R^2}{r^4}\left[\frac{z}{r}\left(\frac{5z^2}{r^2} - 3\right)\right]
\end{aligned}$$

In Eq. (2), the Earth's radius and the distance from the Earth's center to the center of the satellites are denoted by R and r, respectively. J_2 is the 2nd zonal harmonics of the Earth and zonal harmonics less than j_2 have been ignored [21]. the position and velocity vector of the satellite at the Earth-centered, Earth-coordinate frame (ECEF) is denoted by $\boldsymbol{X} = [x, y, z]$ and $\boldsymbol{V} = [\dot{x}, \dot{y}, \dot{z}]$, respectively. As an example, assume that the classic orbital elements of the initial and the final orbit are given in table 1. The orbit's semi-major axis, eccentricity, inclination, right ascension of the ascending node, and the argument of perigee are denoted by a, e, i, Ω, ω.

Table 1: the classic orbital elements of the initial and the final orbit

Orbit	$a(km)$	e	$i(deg)$	Ω(deg)	$\omega(deg)$
Initial	8000	0.25	25	25	8
Final	21500	0.09	25	25	8

Assume that the true anomaly of the arrival points θ_a are given in the mission profile as: $\theta_a = 60^\circ$. The flight time of the mission t and departure θ_d are selected as design parameters and selected by the optimization algorithm. This paper suggested the Particle Swarm Optimization (PSO) algorithm as the optimization algorithm and the Eq. (2) is selected as an objective function that could minimize the total velocity change of the maneuver and consequently the fuel consumption. The process of the PSO is fully described in section 3. The LTP is established in Fig. (1) once due to unperturbed and once to perturbed orbital equations of motion. Note that by setting the perturbation terms to zero in Eq. (2), the unperturbed equations will be obtained.

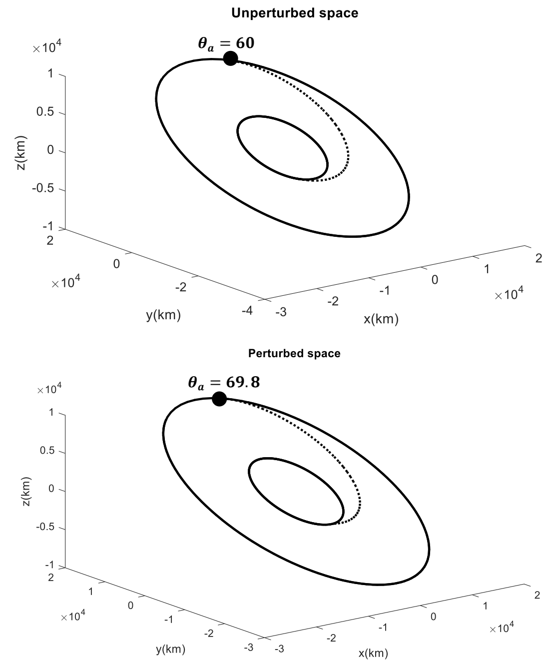

Fig. 1. The LTP maneuver once due to unperturbed and once to perturbed orbital equations of motion

As mentioned the desired true anomaly was $\theta_a = 60^\circ$. But according to Fig. 1, the effect of oblate perturbations has caused the satellite to arrive at the final orbits in $\theta_a = 69.8^\circ$. Therefore, according to Fig. 1, the inability of the LTP to consider perturbations has caused a significant decrease in targeting accuracy. Therefore, this paper proposed an approach to modify the LTP to consider space perturbations and accurate targeting. In section 3, the PSO optimization algorithm will be described, and section 4 will deal with the development of LTP Correction (LTPC).

3. PSO optimization

In computational science, particle swarm optimization (PSO) is a computational method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. First, a finite number of seeds is scattered randomly in the D-dimensional search space, representing the optimization problem's initial solutions. Then according to the seeds' fitness value, the member of the colonies is ranked and sorted. The *i*-th plant, which represents the *i*-th initial position, is shown by $X^{i,best}$. An $X^{i,best}$ which has the most satisfying fitness value is the best position of the colony, and is shown by X^{gbest}. Updating the speed and position of each particle is obtained from the following equations [22]:

$$v^{i,s}[t+1] = \gamma V^i[t] + c_1 r_{1,s}(X^{i,best}[t] - X^i[t]) + c_2 r_{2,s}(X^{gbest}[t] - X^i[t]) \quad (3)$$

$$X^{i,s}[t+1] = X^i[t] + V^{i,s}[t+1] \quad (4)$$

The inertia weight γ may help to keep the motion of each particle in its main direction and balance the global and local search. Also, in Eqs. (3) and (4), the velocity and position of the s-th seed of the i-th member shows by $V^{i,s}$, $X^{i,s}$ respectively. The cognitive coefficient $c_1$and social coefficient $c_2$control the rate of convergence towards a personal best or global best location. $r_{1,s}$ and $r_{2,s}$ are the random number in the range [0,1]. The flight time of the mission t and departure θ_d are selected as design parameters and selected by the PSO optimization algorithm.

4. Lambert Targeting Problem Correction (LTPC)

in this article, an innovative correction algorithm is proposed to modify LTP. The proposed LTPC (LTP Correction) algorithm is developed by combining the LTP with the shooting method. The shooting method solves a two-boundary value problem by reducing it to an initial value problem system[14]. The orbital transfer mission is performed once applying the LTP. Then, the following iterative process is established to modify LTP. the newly developed algorithm is called LTPC, which modifies LTP to consider space perturbations:

1-The LTP is applied and optimized with the PSO for the satellite deployment mission.

2-The departure position vector of the LTP maneuver $\vec{r}_d$ is chosen as an initial condition.

3-The new impulse velocity vector $\vec{V}_{P_s}$ at $\vec{r}_d$ and the new flight time t_s are also selected as an initial condition by PSO, and the shooting algorithm started (starting LTP correction).

4-The transfer orbit is determined by applying the system of Eq. (2) with the selected initial conditions in the previous steps.

5-Calculate the difference between the new position and the velocity vector of the satellite $(\vec{r}_{a_N}, \vec{v}_{a_N})$ relative to the values determined by the LTP $(\vec{r}_a, \vec{v}_a)$ at the arrival point.

6-The LTP is replaced by the transfer orbit determined by the shooting algorithm (LTPC) if condition (5) is satisfied:

$$\left|\vec{r}_{a_N} - \vec{r}_{a_{MILTP}}\right| < 2 \times 10^{-3}\ (m)$$
$$\left|\vec{v}_{a_N} - \vec{v}_{a_{MILTP}}\right| < 10^{-3}\ \left(\frac{km}{s}\right) \quad (5)$$

7-If the delta values are not met condition (5), then return to step 3 and shoot again.

In the results section, the correctness of the proposed algorithm will be validated

5. Result

The main peruse of this article is to develop an orbital transfer that has accurate targeting and could regard space perturbation. For this reason, first, the LTP is suggested for accurate targeting. Since the LTP cannot consider space perturbations, this paper proposed a correction on the LTP that modified the LTP to regard space perturbations with accurate targeting. In this section, two cases study is investigated.

The classic orbital elements of the initial and the final orbit of these cases study are given in table 2.

Table 2: the classic orbital elements of the initial and the final orbit of the case study.

Orbit	$a(km)$	e	$i(deg)$	Ω(deg)	$\omega(deg)$
Initial	9000	0.1	20	20	10
Final	23000	0.08	20	20	10

In the first case study, it is assumed that the satellite must be deployed at $\theta_a = 150^\circ$. The LTP is compared with the proposed LTPC in Fig. 2.

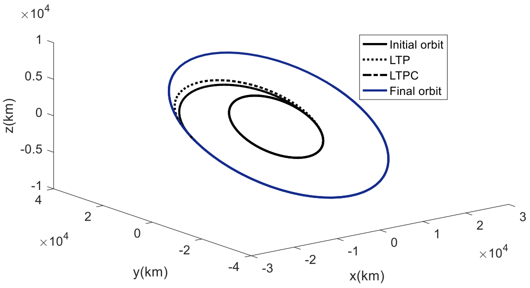

Fig. 2. The comparison between the LTP and proposed LTPC in the first case study

The arrival true anomaly θ_a, the total velocity change ΔV_{tot} and the flight time t between the LTP and the proposed LTPC are compared in table 3 for the first case study.

Table 3: the comparison of arrival true anomaly θ_a, the total velocity change ΔV_{tot} and the flight time t between the LTP and the proposed LTPC

	$\theta_a(deg)$	$\Delta V_{tot}\left(\frac{km}{s}\right)$	$t(sec)$
LTP	161	2.18	10403
LTPC	153	2.21	10487

According to the table 2, the delta true anomaly between the desired true anomaly $\theta_a = 150^\circ$ and the arrival true anomaly LTP and proposed LTPC are 11° and 3° respectively. Therefore, the LTPC has accurate targeting with considering space perturbation. This is despite the fact that the difference between the velocity change and flight time of the LTP and LTP are neglectable.

In the first case study it is assumed that the satellite must be deployed at $\theta_a = 225^{\circ}$. The LTP is compared with the proposed LTPC in the Fig. 3.

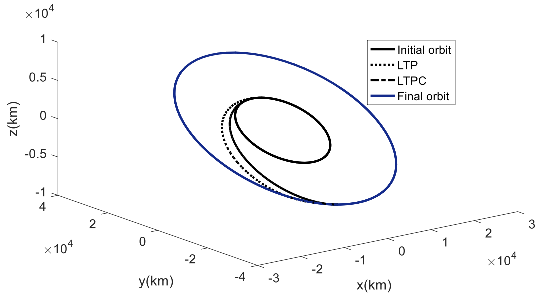

Fig. 3. The comparison between the LTP and proposed LTPC in the second case study

The arrival true anomaly θ_a, the total velocity change ΔV_{tot} and the flight time t between the LTP and the proposed LTPC are compared in table 4 for the first case study.

Table 4: the comparison of arrival true anomaly θ_a, the total velocity change ΔV_{tot} and the flight time t between the LTP and the proposed LTPC

	$\theta_a(deg)$	$\Delta V_{tot}\left(\frac{km}{s}\right)$	$t(sec)$
LTP	237.4	2.32	11812
LTPC	227.5	2.35	11854

According to table 4, the delta true anomaly between the desired true anomaly $\theta_a = 225$ and the arrival true anomaly LTP and proposed LTPC are 12.4° and 2.5°respectively. Therefore, like the first case study, the LTPC has accurate targeting considering the space perturbation and as illustrated in the first case study although the difference between the velocity change and flight time of the LTP and LTP are neglectable.

6. Conclusion

The main purpose of this investigation was to introduce an orbital transfer maneuver that has accurate targeting and could consider every space perturbation that made the mission practical. The LTP was proposed first as an orbit transfer maneuver that had accurate targeting. Since the LTP cannot consider space perturbation so the accuracy of the targeting was reduced significantly and due to the following disadvantages it was not an appropriate maneuver for the operational mission

- LTP could not consider space perturbations due to the use of Kepler's equations of motion.
- Due to the accumulative trait of the perturbations, LTP was not suitable for long-term missions.

In this article, an innovative correction algorithm was proposed to modify LTP. The proposed LTPC (LTP Correction) algorithm was developed by combining the LTP with the shooting method .The orbital transfer mission is performed once applying the LTP. Then, the five consecutive stages were iteratively established to modify LTP. the newly developed algorithm is called LTPC, which modifies LTP to consider space perturbations.

The Earth's oblateness was regarded as a perturbation of interest. The results indicated that the delta true anomaly between the desired arrival true anomaly and the arrival true anomaly of the proposed LTPC was much less than the delta true anomaly between the desired arrival true anomaly and the arrival true anomaly of the classical LTP. This is while there is no significant difference between the flight time and total requirement velocity change between LTP and the proposed LTPC. Therefore, the proposed LTPC had a much higher targeting accuracy, which is due to the ability to consider space perturbations

7. References:

[1] J. J. Gray, “Johann Heinrich Lambert, mathematician and scientist, 1728–1777,” *Hist. Math.*, vol. 5, no. 1, pp. 13–41, 1978.

[2] J. Kriz, “A uniform solution of the Lambert problem,” *Celest. Mech.*, vol. 14, no. 4, pp. 509–513, 1976.

[3] A. Sukhanov and A. F. Prado, “Lambert problem solution in the hill model of motion,” *Celest. Mech. Dyn. Astron.*, vol. 90, no. 3, pp. 331–354, 2004.

[4] H. Leeghim and B. A. Jaroux, “Energy-optimal solution to the lambert problem,” *J. Guid. Control. Dyn.*, vol. 33, no. 3, pp. 1008–1010, 2010.

[5] R. M. Woollands, A. Bani Younes, and J. L. Junkins, “New solutions for the perturbed lambert problem using regularization and picard iteration,” *J. Guid. Control. Dyn.*, vol. 38, no. 9, pp. 1548–1562, 2015.

[6] B. F. Thompson and L. J. Rostowfske, “Practical constraints for the applied Lambert problem,” *J. Guid. Control. Dyn.*, vol. 43, no. 5, pp. 967–974, 2020.

[7] P. V Arlulkar and S. D. Naik, “Solution based on dynamical approach for multiple-revolution lambert problem,” *J. Guid. Control. Dyn.*, vol. 34, no. 3, pp. 920–923, 2011.

[8] Q. He, J. Li, and C. Han, “Multiple-revolution solutions of the transverse-eccentricity-based lambert problem,” *J. Guid. Control. Dyn.*, vol. 33, no. 1, pp. 265–269, 2010.

[9] N. Adurthi and M. Majji, “Uncertain lambert problem: a probabilistic approach,” *J. Astronaut. Sci.*, vol. 67, no. 2, pp. 361–386, 2020.

[10] Q. M. Al-Mdallal, M. I. Syam, and M. N. Anwar, “A collocation-shooting method for solving fractional boundary value problems,” *Commun. Nonlinear Sci. Numer. Simul.*, vol. 15, no. 12, pp. 3814–3822, 2010.

[11] S. N. Ha, “A nonlinear shooting method for two-point boundary value problems,” *Comput. Math. with Appl.*, vol. 42, no. 10–11, pp. 1411–1420, 2001.

[12] H. Hajimirsadeghi and C. Lucas, “A hybrid IWO/PSO algorithm for fast and global optimization,” in *IEEE EUROCON 2009*, 2009, pp. 1964–1971.
[13] M. Bakhtiari, K. Daneshjou, and E. Abbasali, “A new approach to derive a formation flying model in the presence of a perturbing body in inclined elliptical orbit: relative hovering analysis,” *Astrophys. Space Sci.*, vol. 362, no. 2, 2017, doi: 10.1007/s10509-016-2968-9.
[14] E. Abbasali, A. Kosari, and M. Bakhtiari, “Effects of oblateness of the primaries on natural periodic orbit-attitude behaviour of satellites in three body problem,” *Adv. Sp. Res.*, 2021.
[15] M. Bakhtiari, E. Abbasali, S. Sabzy, and A. Kosari, “Natural coupled orbit—attitude periodic motions in the perturbed-CRTBP including radiated primary and oblate secondary,” *Astrodynamics*, vol. 7, no. 2, pp. 229–249, 2023.
[16] E. Abbasali and M. Bakhteiari, “Restricted three body problem considering the perturbations of both oblate massive primaries,” *J. Aerosp. Sci. Technol.*, vol. 13, no. 2.
[17] A. Shirazi, J. Ceberio, and J. A. Lozano, “Optimal multi-impulse space rendezvous considering limited impulse using a discretized Lambert problem combined with evolutionary algorithms,” 2019.
[18] A. Shirazi, J. Ceberio, and J. A. Lozano, “An evolutionary discretized Lambert approach for optimal long-range rendezvous considering impulse limit,” *Aerosp. Sci. Technol.*, vol. 94, p. 105400, 2019.
[19] D. A. Vallado, *Fundamentals of astrodynamics and applications*, vol. 12. Springer Science & Business Media, 2001.
[20] H. D. Curtis, *Orbital mechanics for engineering students*. Butterworth-Heinemann, 2013.
[21] M. Bakhtiari, K. Daneshjou, and M. Fakoor, “Relative hovering analysis about an elliptical perturbed orbit with consideration of dynamic air drag and oblate earth,” *Aerosp. Sci. Technol.*, vol. 70, pp. 286–299, 2017.
[22] M. Bakhtiari, K. Daneshjou, and A. A. Mohammadi-Dehabadi, “The effects of parking orbit elements on designing of on-orbit servicing missions,” *Proc. Inst. Mech. Eng. Part G J. Aerosp. Eng.*, vol. 233, no. 3, pp. 793–810, 2019.