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# Modeling Electricity Demand: A Comparative Study of Univariate, Multivariate, and Functional SSA

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**Abstract**

Accurate modeling of patterns in electricity demand is essential for effective energy forecasting and grid management. This study presents a comparative analysis of Singular Spectrum Analysis (SSA) in its univariate (SSA), multivariate (MSSA), and functional (FSSA) forms, applied to half-hourly electricity demand data from the state of Victoria, Australia. The data, exhibiting pronounced daily and weekly seasonality, is reshaped into univariate, multivariate, and functional time series formats to align with the respective SSA frameworks.

Each method is evaluated across a range of window lengths and grouping strategies, with performance assessed using the Root Mean Square Error (RMSE) of the reconstructed series. Our results show that Univariate SSA consistently yields the most accurate reconstructions, followed by MSSA, while FSSA exhibits higher reconstruction error in this setting. Additionally, we investigate the seasonal components extracted by each method and find that while MSSA and FSSA effectively capture weekly periodicity, only Univariate SSA is capable of detecting both daily and sub-daily (e.g., hourly) patterns—albeit requiring careful selection of window length.

These findings highlight the importance of choosing the appropriate SSA variant based on the frequency characteristics of the data and the specific goals of analysis, especially when dealing with high-resolution time series.

**Keywords:** Singular Spectrum Analysis (SSA); Functional Time Series; Electricity Demand Forecasting; Seasonality; Time Series Decomposition.
**Mathematics Subject Classification (2020):** 62M10, 62H12, 62P30.

## 1 Introduction

Electricity demand exhibits pronounced seasonal patterns driven by climatic conditions, economic activity, and social behavior. Accurately modeling and forecasting this seasonality is

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critical for energy planning, infrastructure development, and operational reliability in power systems. Traditional time series models have long been employed for this purpose; however, recent advances in multivariate and functional time series (FTS) methods (Hyndman and Athanasopoulos (2018) and Amato and *et al.* (2025)) offer new avenues for capturing complex temporal dependencies and structural variations in electricity consumption data.

The complex nature of time series, which often violates the assumptions of traditional time series analysis methods, motivates the introduction of new approaches and the development of the literature. Among these, we focus on Singular Spectrum Analysis (SSA), a non-parametric and model-free method for time series analysis that has been developed for univariate (Golyandina and *et al.* (2001)), multivariate (Hassani and Mahmoudvand (2013)), and functional (Haghbin and *et al.* (2021)) time series.

This paper presents a comparative study of univariate, multivariate, and functional SSA for modeling the seasonal dynamics of electricity demand. Each framework provides distinct advantages: univariate models are computationally efficient and interpretable, multivariate models capture interdependencies among related series, and FTS models leverage the continuous nature of daily or intra-daily load profiles to preserve functional characteristics. By applying these techniques to real-world electricity demand data, we assess their relative performance in capturing seasonal structures and producing accurate forecasts.

Our study contributes to the growing body of literature on energy demand forecasting by highlighting the trade-offs and practical considerations associated with different modeling strategies. The findings offer insights for researchers and practitioners seeking robust and scalable tools for seasonal demand analysis in increasingly complex energy systems.

# 2 Methodology

In this section, we provide a brief introduction to the methodology of model capturing in SSA. As it has mentioned above, we have three different version of SSA that admit different object as input. However, the general algorithm for all version are the same.

## 2.1 SSA decomposition for univariate time series

Let $\{y_1, \ldots, y_N\}$ denote a univariate time series of length $N$. SSA decomposition begins by transforming the time series into a structured matrix form, followed by matrix decomposition and reconstruction steps that isolate distinct components such as trend, oscillations, and noise. The overall process is outlined below.

1. **Embedding:** Choose a window length parameter $L$ such that $2 \leq L \leq N-1$, and define $K = N - L + 1$. Construct the trajectory (Hankel) matrix $\mathbf{Y} \in \mathbb{R}^{L \times K}$, where each column is a lagged segment of the original series:

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_K \\ y_2 & y_3 & \cdots & y_{K+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_L & y_{L+1} & \cdots & y_N \end{bmatrix}$$

2. **Singular Value Decomposition (SVD):** Decompose the matrix $\mathbf{Y}$ using SVD into a sum of rank-one matrices: $\mathbf{Y} = \mathbf{Y}_1 + \cdots + \mathbf{Y}_d$.

3. **Grouping:** Partition the set of elementary matrices into $m$ disjoint groups $I_1, \ldots, I_m$, and form grouped matrices by summing the elements within each group. In the simplest case ($m = 2$), group $I_1 = 1, \ldots, r$ is associated with the signal (e.g., trend and periodicity), and $I_2 = r+1, \ldots, d$ corresponds to noise.

4. **Diagonal Averaging (Reconstruction):** Transform each grouped matrix $\mathbf{Y}_{I_j}$ back into a time series of length $N$ via diagonal averaging (also known as Hankelization). This step reconstructs the components of the original series.

## 2.2 SSA decomposition for the Multivariate Time Series

Let $\{\mathbf{y}_1, \ldots, \mathbf{y}_N\}$ where $\mathbf{y}_t \in \mathbb{R}^D$, is a $D$-variate time series of length $N$. Multivariate SSA (MSSA) follows a similar procedure to univariate SSA, with the primary difference being in the construction of the trajectory matrix. Specifically, the trajectory matrix is formed by horizontally stacking the trajectory matrices of each individual (marginal) time series.
In this case, the combined trajectory matrix $\mathbf{Y}$ is defined as:

$$\mathbf{Y} = \left[\mathbf{Y}^{(1)}\ \mathbf{Y}^{(2)}\ \cdots\ \mathbf{Y}^{(D)}\right],$$

where each $\mathbf{Y}^{(i)}$ is the trajectory matrix corresponding to the $i^{\text{th}}$ time series. The rest of the SSA procedure (SVD, grouping, diagonal averaging, etc.) is then applied to this combined matrix.

## 2.3 SSA decomposition for the Functional Time Series

1. **Embedding:** Given a functional time series $\{X_t(u)\}_{t=1}^N$ in a Hilbert space $\mathcal{H}$, construct the trajectory matrix:

$$\mathbf{Y} = [X_1(u), X_2(u), \ldots, X_L(u)] \in \mathcal{H}^L$$

   where $L$ is the window length such that $1 < L < N$.

2. **Decomposition:** Apply singular value decomposition (SVD) to the trajectory matrix:

$$\mathbf{Y} = \sum_{i=1}^{L} \sqrt{\lambda_i} U_i \otimes V_i$$

   where $\lambda_i$ are the eigenvalues, and $U_i$, $V_i$ are the corresponding orthonormal singular functions.

3. **Grouping:** Partition the set of indices $\{1, 2, \ldots, L\}$ into $d$ disjoint groups $I_1, I_2, \ldots, I_d$, according to the desired signal components (e.g., trend, seasonality, noise).

4. **Reconstruction:** For each group $I_k$, form the grouped approximation:

$$\mathbf{Y}^{(k)} = \sum_{i \in I_k} \sqrt{\lambda_i} U_i \otimes V_i$$

   and apply diagonal averaging (also known as Hankelization) to reconstruct the functional time series component $\{\tilde{X}_t^{(k)}(u)\}_{t=1}^N$.

# 3 Empirical analysis

## 3.1 Data description

The Half-hourly electricity demand dataset, available in the `fpp2` package in R, contains electricity demand data for the state of Victoria, Australia. The dataset spans from 1 January 2014 to 31 January 2014, with measurements recorded every 30 minutes, resulting in 48 observations per day. Figure 1 illustrates three perspectives of this dataset: as a univariate time series, a multivariate time series (where each of the 48 half-hour intervals is treated as a separate variable), and as a functional time series (where each day is represented by a continuous function over time).

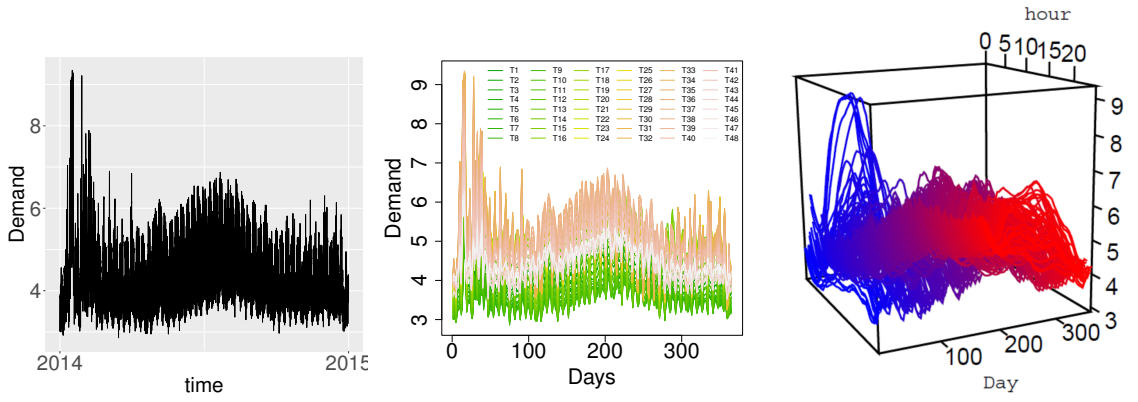



Figure 1: Univariate (left), Multivariate (middle), and Functional (right) representations of the Half-hourly electricity demand dataset.

As evident in Figure 1 and due to the intrinsic daily cycle of electricity usage, the data exhibits a strong seasonal pattern. As discussed in Golyandina and *et al.* (2001) and supported by other studies, the window length in SSA can be chosen as a multiple of the seasonal period, rather than half the series length—especially when dealing with long series where computation becomes intensive.

## 3.2 Model fitting

To evaluate the performance of different SSA frameworks, we applied Univariate SSA (SSA), Multivariate SSA (MSSA), and Functional SSA (FSSA) on the dataset. We experimented with window lengths ranging from 24 to 60 and grouping choices varying from 3 to 8. The RMSE for the reconstructed signals was calculated and used to compare performance across methods. Figure 2 displays the RMSE results. Among the three approaches, SSA yields the best performance, followed by MSSA. FSSA, in this case, resulted in the highest RMSE, indicating comparatively poorer reconstruction accuracy.

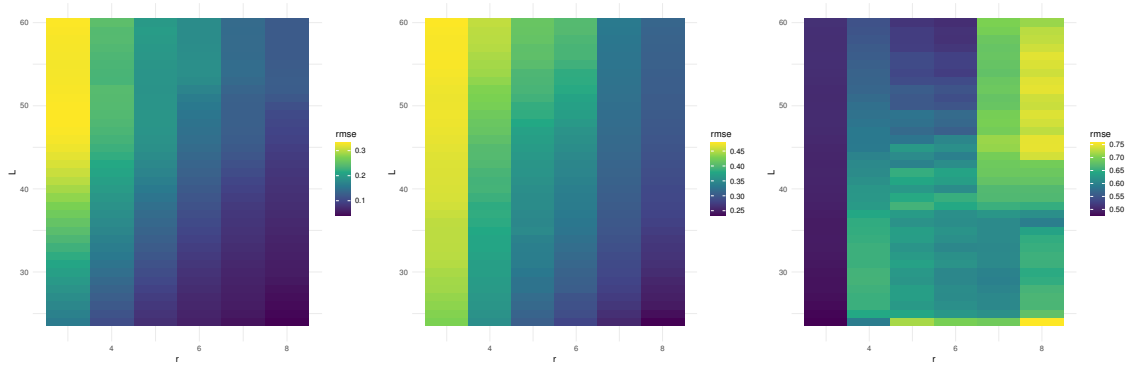



Figure 2: RMSE for fitting Univariate (left), Multivariate (middle), and Functional (right) SSA models to the Half-hourly electricity demand dataset.

## 3.3 Seasonality analysis

As has been established, the inputs for univariate SSA, multivariate SSA, and functional SSA differ, implying that the objective of the analysis must be clearly defined to enable a fair comparison. In this study, we focus on the fitting performance, which is one of the most common objectives in time series modeling. In this section, we explore the seasonal components identified by the aforementioned methods.

We fixed the window length at $L = 48$, although we tested other values and obtained similar results. As illustrated in Figures 3 and 4, multivariate SSA and functional SSA reveal a clear weekly seasonality (7-day cycle). Univariate SSA, on the other hand, identifies two dominant seasonal components with periods of approximately 48 and 24, corresponding to daily and half-daily cycles, respectively.

It is important to note that the time unit in the multivariate and functional SSA representations is the day, while in the univariate SSA, it is the half-hour. Therefore, capturing a 7-day periodicity in the univariate SSA requires detecting a periodic component at lag 336. We verified that such periodicity is indeed captured when the window length exceeds 336. However, identifying it via paired eigentriple plots is challenging, so we employed the `parestimate` function from the `Rssa` package for more precise period estimation. From this perspective, multivariate SSA and functional SSA offer more interpretable insights into weekly seasonality.

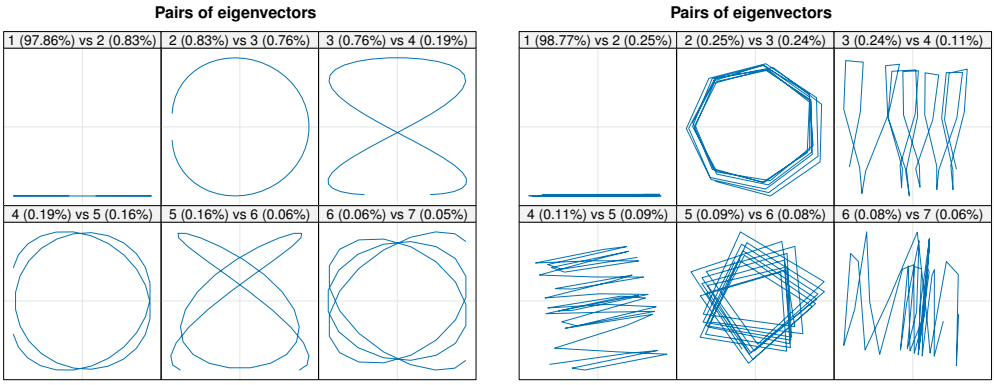



Figure 3: Paired plot of eigenvectors using the univariate SSA (left) and multivariate SSA (right).

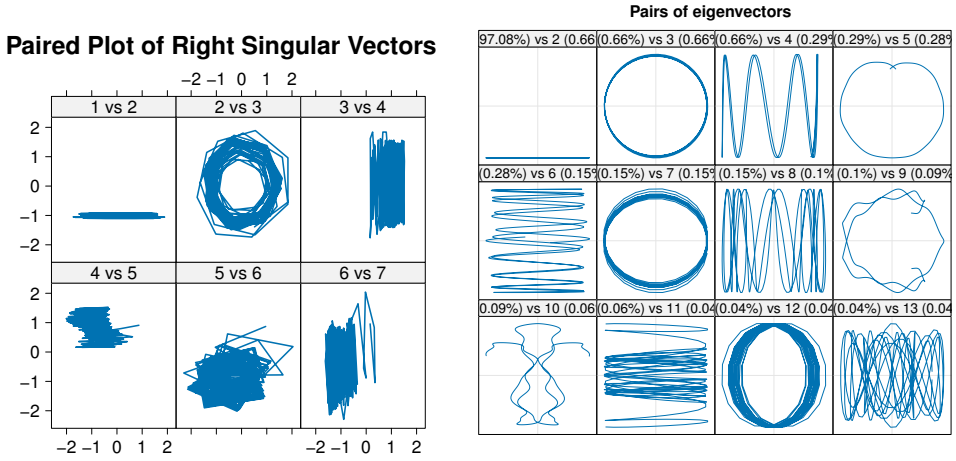



Figure 4: Paired plot of right singular vectors using the Functional SSA (left) and paired plot of eigenvectors using univariate SSA with larger window length (right).

# 4 Conclusion

In this study, we investigated the performance of Univariate Singular Spectrum Analysis (SSA), Multivariate SSA (MSSA), and Functional SSA (FSSA) on half-hourly electricity demand data. By exploring a range of window lengths and grouping structures, we assessed the reconstruction accuracy of each method using the Root Mean Squared Error (RMSE) as the evaluation metric. The results indicate that Univariate SSA consistently outperforms both MSSA and FSSA in terms of fitting accuracy.

Furthermore, we examined the seasonal components captured by each approach. Our findings show that while multivariate and functional SSA are effective at identifying weekly seasonality (e.g., 7-day periodicity), they are unable to capture sub-daily patterns such as hourly or half-daily periodicities. In contrast, Univariate SSA successfully detects both daily and sub-daily seasonalities. However, identifying such components requires careful tuning of the window length, which may not be straightforward in practice.

These findings underscore the importance of selecting the appropriate SSA variant based on the temporal resolution of the data and the specific goals of the analysis. While MSSA and FSSA offer interpretability for coarser seasonal structures, Univariate SSA remains the most flexible and powerful tool for uncovering fine-grained periodicities in high-resolution time series.

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