پانزدهمین سمینار احتمال
و فرآیندهای تصادفی
۸ و ۹ شهریور ۱۴۰۴
دانشگاه کردستان

15th Seminar On Probability and Stochastic Processes
August 30-31, 2025
University of Kurdistan

انجمن آمار ایران

# European Option Pricing under Market Uncertainty and Liquidity Risk: A Heston-Based Stochastic Volatility Model

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**Abstract**

This paper addresses the study of European options in markets where the underlying asset is subject to liquidity-related risks. In the absence of liquidity effects, the asset price dynamic is governed by the Heston stochastic volatility model. Liquidity risk is introduced through market liquidity as a stochastic process, which adjusts the asset price via a liquidity discount mechanism. By applying a change of measure, we derive the dynamics of the model under the risk-neutral framework. An explicit form of the characteristic function for the logarithm of the asset price is then obtained, enabling the derivation of a closed-form solution for European option prices. The numerical example and analysis of the sensitivity of key parameters confirm the theoretical results.

**Keywords:** Option Pricing, Heston Model, Liquidity.

**Mathematics Subject Classification (2020):** 65CXX, 60HXX.

## 1 Introduction

Following the foundational work of Black and Scholes (1973), which modeled asset prices using normally distributed log-returns, subsequent models have sought to address more realistic market dynamics. These advancements introduced multiple stochastic factors, such as classical stochastic volatility frameworks Heston (1993) and Stein and Stein (1991), regime-switching dynamics He and Lin (2023), multifactor volatility specifications Cortazar and *et al.* (2009) and El-Khatib and *et al.* (2024) , and hybrid models incorporating both stochastic volatility and interest rates Grzelak and Oosterlee (2011). Among these, the Heston model Heston (1993) has remained widely adopted due to its analytical tractability and its capacity to capture essential volatility features like mean reversion and non-negativity. Although many existing models offer significant strengths, they often overlook the essential role of liquidity risk—an aspect widely recognized as critical in real-world financial risk management. Incorporating liquidity considerations into derivative pricing frameworks remains a necessary yet underdeveloped area. Despite its practical importance to both market practitioners and risk

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professionals, research on this topic is still emerging, with no broadly accepted framework established to date. Liquidity-aware option pricing models vary Li and *et al.* (2018) and Loeper (2018), with some focusing on trader impact Leippold and Schärer (2017) and Ludkovski and Shen (2013), others using bid-ask spreads Zhang and *et al.* (2019), and some applying price discounts based on trading speed during illiquid periods Subramanian and Jarrow (2001) and Longstaff and *et al.* (2005). Growing empirical evidence highlights the importance of market liquidity in shaping asset prices, motivating its integration into stock price dynamics Feng and *et al.* (2016). Madan and Cherny (2010) introduced a framework treating the market as a counterparty and modeling liquidity through a stress parameter, later refined to better capture its effect on returns Albrecher and Schoutens (2013) and Corcueraand and *et al.* (2010). This led to derivative pricing models that discount stock prices based on liquidity conditions Caldarera and *et al.* (2005). Feng and *et al.* (2014) advanced this by introducing stochastic market liquidity, which has since been applied to pricing various options, including European Puneet and *et al.* (2022), American Zhang and *et al.* (2019), and Asian options Li and *et al.* (2018). To address the limitations of prior models that rely on the Black-Scholes framework, we propose an option pricing model combining Heston's stochastic volatility with a liquidity discount factor. Despite involving four stochastic drivers, we derive a closed-form solution for European options using a benchmark transformation. Numerical results highlight the effects of both liquidity and volatility risks, supported by sensitivity analysis.

The structure of the paper is as follows. Section 2 introduces the proposed option pricing framework under the real-world measure. In Section 3, we apply a transforming the physical measure into an equivalent martingale framework and present the pricing closed-form formula. Section 4 offers numerical results, and final remarks are provided in the concluding section.

## 2 Option pricing based on the Heston volatility model

We consider a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and adopt the Heston stochastic volatility model given by:

$$\begin{aligned} \frac{dP_t^H}{P_t^H} &= \mu\, dt + \sqrt{v_t}\, dB_{1,t}, \\ dv_t &= \eta(\theta - v_t)\, dt + \sigma\sqrt{v_t}\, dB_{2,t} \end{aligned} \tag{1}$$

where $P_t^H$ denotes the stock price, $\mu$ is the expected return, $v_t$ represents the instantaneous variance, $\theta$ is the long-run mean of the variance, $\eta$ is the rate of mean reversion, $\sigma$ is the volatility of volatility, and $\{B_{i,t}\}_{t\geq 0}$ for $i = 1, 2$ are standard Brownian motions with $dB_{1,t}dB_{2,t} = \rho_1 dt$. It is important to emphasize that the dynamics described in equation (1) are not directly suitable for derivative pricing, as real-world stock markets often experience imbalances in supply and demand, resulting in liquidity issues. Therefore, accurately modeling liquidity risk within stock dynamics demands careful and deliberate treatment.

Following Black and Scholes (1973) and Feng and *et al.* (2014), we model liquidity risk through supply-demand interaction, assuming a constant stock supply $\bar{P}$, which adjusts prices in response to market demand fluctuations. The demand is modeled by the function $d(P_t, \delta_t, \epsilon_t)$, defined as

$$d(P_t, \delta_t, \epsilon_t) = \left(\frac{P_t \epsilon_t}{\omega \delta_t}\right) \Upsilon, \tag{2}$$

where $Z(\cdot)$ is a smooth, strictly increasing function that captures how demand responds to price, liquidity sensitivity $\delta_t$, and an external factor $\epsilon_t$. In this framework, the observed stock price $P_t$ incorporates liquidity risk by discounting the Heston price $P_t^H$ via a stochastic discount factor $\epsilon_t$:

$$P_t = \frac{1}{\epsilon_t} P_t^H. \tag{3}$$

The liquidity-adjusted price is derived from market equilibrium:

$$\left(\frac{\delta_t \omega}{\epsilon_t P_t}\right) \Upsilon = \bar{P}, \quad \Rightarrow \quad P_t = \frac{1}{\epsilon_t} \cdot \frac{\delta_t \omega}{\Upsilon^{-1}(\bar{P})}. \tag{4}$$

When liquidity is perfect ($\epsilon_t = 1$), the price reduces to:

$$P_t^H = \frac{\delta_t \omega}{Z^{-1}(\bar{P})}. \tag{5}$$

The liquidity discount factor $\epsilon_t$ evolves as a geometric Brownian motion:

$$\frac{d\epsilon_t}{\epsilon_t} = \left(\frac{1}{2}\lambda^2 M_t^2 - \lambda M_t\right) dt - \lambda M_t dB_{3,t}, \tag{6}$$

with its explicit solution:

$$\epsilon_t = \exp\left(-\lambda \int_0^t m_s \, ds + \lambda \int_0^t m_s \, dB_{3,s}\right). \tag{7}$$

Applying Itô's product rule gives the dynamics of the liquidity-adjusted stock price:

$$\frac{dP_t}{P_t} = \left(\mu + \lambda M_t + \frac{1}{2}\lambda^2 M_t^2\right) dt + \sqrt{v_t} dB_{1,t} + \lambda M_t dB_{3,t}. \tag{8}$$

The market liquidity $M_t$ follows an Ornstein–Uhlenbeck process:

$$dM_t = \alpha(\beta - M_t)\, dt + \delta \, dB_{4,t}, \tag{9}$$

where $B_{4,t}$ is a Brownian motion with $dB_{1,t}dB_{4,t} = 0$, $dB_{2,t}dB_{4,t} = 0$, and $dB_{3,t}dB_{4,t} = \rho_2$. Decoupling liquidity and price dynamics simplifies analysis and remains financially valid, despite reduced model flexibility from not using full correlation.

This model captures the dual impact of stochastic volatility and liquidity risk, forming a two-factor pricing framework.

# 3 European option pricing

To enable European option pricing within the liquidity-adjusted Heston model, an equivalent martingale measure must first be established, as the current model is defined under the physical measure.

## 3.1 A risk-adjusted probability measure

In a market where both stochastic volatility and liquidity risk are involved, the system is inherently incomplete due to the presence of multiple sources of uncertainty. Consequently, there is no unique risk-neutral measure, and one must be selected under suitable conditions.

To handle correlations among the driving Brownian motions, the stock price process is reformulated as:

$$\frac{dP_t}{P_t} = \left(\mu + \lambda M_t + \frac{1}{2}\lambda^2 M_t^2\right) dt + \sqrt{v_t}(\rho_1 dB_{2,t} + \sqrt{1-\rho_1^2} dB_{5,t}) + \lambda M_t(\rho_2 dB_{4,t} + \sqrt{1-\rho_2^2} dB_{6,t}), \tag{10}$$

where $B_{2,t}, B_{4,t}, B_{5,t}, B_{6,t}$ are independent Brownian motions. We define the Radon-Nikodym derivative for transitioning from the physical measure $\mathbb{P}$ to a risk-neutral measure $\mathbb{Q}$:

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left(-\sum_{i=1}^{4} \int_0^t \eta_{i,s} dB_{i,s} - \frac{1}{2}\int_0^t \eta_{i,s}^2 ds\right), \tag{11}$$

which yields the following transformed Brownian motions under $\mathbb{Q}$:

$$\begin{aligned} dB_{2,t}^{\mathbb{Q}} &= dB_{2,t} + \eta_{1,t} dt, \\ dB_{4,t}^{\mathbb{Q}} &= dB_{4,t} + \eta_{2,t} dt, \\ dB_{5,t}^{\mathbb{Q}} &= dB_{5,t} + \eta_{3,t} dt, \\ dB_{6,t}^{\mathbb{Q}} &= dB_{6,t} + \eta_{4,t} dt. \end{aligned} \tag{12}$$

Assuming that the market prices of volatility and liquidity risks are proportional to their respective levels, we let:

$$\sigma\sqrt{v_t}\eta_{1,t} = \eta_1 v_t, \quad \delta\eta_{2,t} = \eta_2 M_t. \tag{13}$$

This leads to the transformed dynamics:

$$\begin{aligned} dv_t &= \bar{\eta}(\bar{\theta} - v_t)dt + \sigma\sqrt{v_t}dB^{\mathbb{Q}}_{2,t}, \\ dM_t &= \bar{\alpha}(\bar{\beta} - M_t)dt + \delta dB^{\mathbb{Q}}_{4,t}, \end{aligned} \tag{14}$$

where the adjusted parameters are:

$$\bar{\eta} = \eta + \eta_1, \quad \bar{\theta} = \frac{\eta\theta}{\eta + \eta_1}, \quad \bar{\alpha} = \alpha + \eta_2, \quad \bar{\beta} = \frac{\alpha\beta}{\alpha + \eta_2}. \tag{15}$$

Substituting the transformed Brownian motions into the stock price process gives:

$$\frac{dP_t}{P_t} = \bar{\mu}dt + \sqrt{v_t}(\rho_1 dB^{\mathbb{Q}}_{2,t} + \sqrt{1-\rho_1^2}dB^{\mathbb{Q}}_{5,t}) + \lambda M_t(\rho_2 dB^{\mathbb{Q}}_{4,t} + \sqrt{1-\rho_2^2}dB^{\mathbb{Q}}_{6,t}), \tag{16}$$

where

$$\bar{\mu} = \mu + \lambda M_t + \frac{1}{2}\lambda^2 M_t^2 - \sqrt{v_t}(\rho_1\eta_{1,t} + \sqrt{1-\rho_1^2}\eta_{3,t}) - \lambda M_t(\rho_2\eta_{2,t} + \sqrt{1-\rho_2^2}\eta_{4,t}). \tag{17}$$

By enforcing the no-arbitrage condition under $\mathbb{Q}$ (i.e., requiring that the discounted stock price is a martingale), we must have $\bar{\mu} = r$, the risk-free rate. Solving for this yields:

$$\mu + \lambda M_t + \frac{1}{2}\lambda^2 M_t^2 - \sqrt{v_t}(\rho_1\eta_{1,t} + \sqrt{1-\rho_1^2}\eta_{3,t}) - \lambda M_t(\rho_2\eta_{2,t} + \sqrt{1-\rho_2^2}\eta_{4,t}) = r. \tag{18}$$

Under the risk-neutral measure, the final form of the stock price SDE becomes:

$$\frac{dP_t}{P_t} = rdt + \sqrt{v_t}dB^{\mathbb{Q}}_{1,t} + \lambda M_t dB^{\mathbb{Q}}_{3,t}, \tag{19}$$

where the volatility and liquidity processes evolve according to the dynamics in Equation (4).

## 3.2 Closed-form formula

With the risk-neutral dynamics established, we now address the pricing of European options where both stochastic volatility and liquidity effects are present. Let $C(P_t, M_t, v_t, t)$ denote the option value at time $t$, with $\mathbb{F}_i^{\mathbb{Q}}$ representing the filtration generated by Brownian motions under the risk-neutral measure $\mathbb{Q}$. The pricing formula becomes:

$$C(P_t, M_t, v_t, t) = \mathbb{E}^{\mathbb{Q}}\left[e^{-r(T-t)}(P_t - K)^+ \,\middle|\, \mathcal{F}^{B^{\mathbb{Q}}_2} \vee \mathcal{F}^{B^{\mathbb{Q}}_4} \vee \mathcal{F}^{B^{\mathbb{Q}}_5} \vee \mathcal{F}^{B^{\mathbb{Q}}_6}\right] = e^{-r(T-t)}(J_1 - KJ_2) \tag{20}$$

Define $x_t = \ln P_t$, then:

$$J_1 = \int_{\ln K}^{\infty} e^{x_T} l(x_T)dx_T, \quad J_2 = \int_{\ln K}^{\infty} l(x_T)dx_T, \tag{21}$$

where $l(x_T)$ is the conditional density function of $x_T$. We utilize the characteristic function

$$f(\phi) \equiv f(\phi; x_t, M_t, v_t, t, T) = \mathbb{E}^{\mathbb{Q}}[e^{j\phi x_T} \,|\, \mathcal{F}^{B^{\mathbb{Q}}_2} \vee \mathcal{F}^{B^{\mathbb{Q}}_4} \vee \mathcal{F}^{B^{\mathbb{Q}}_5} \vee \mathcal{F}^{B^{\mathbb{Q}}_6}] = \int_{-\infty}^{+\infty} e^{j\phi x_T} l(x_T) dx_T \tag{22}$$

to express:

$$J_1 = f(-j)\left(\frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \Re\left(\frac{e^{-j\phi\ln K}}{j\phi}\frac{f(\phi - j)}{f(-j)}\right)d\phi\right), \tag{23}$$

$$J_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \Re\left(\frac{e^{-j\phi\ln K}}{j\phi}f(\phi)\right)d\phi. \tag{24}$$

The characteristic function satisfies the PDE:

$$\frac{\partial f}{\partial t} + \frac{1}{2}(\lambda^2 m^2 + v)\frac{\partial^2 f}{\partial x^2} + \frac{1}{2}\sigma^2 v\frac{\partial^2 f}{\partial v^2} + \frac{1}{2}\delta^2\frac{\partial^2 f}{\partial m^2} + \rho_1\sigma v\frac{\partial^2 f}{\partial x\partial v} + \rho_2\lambda\delta m\frac{\partial^2 f}{\partial x\partial m}$$

$$+\left(r-\frac{1}{2}(\lambda^2 m^2+v)\right)\frac{\partial f}{\partial x}+\bar{\eta}(\bar{\theta}-v)\frac{\partial f}{\partial v}+\bar{\alpha}(\bar{\beta}-m)\frac{\partial f}{\partial m}=0, \tag{25}$$

with terminal condition: $f(\phi; x_T, M_t, v_T, T) = e^{j\phi x_T}$. Assuming an affine form:

$$f(\phi)=\exp\left(I_1(\phi)+I_2(\phi)m+I_3(\phi)m^2+I_4(\phi)v+j\phi x\right), \tag{26}$$

leads to ODEs for $I_1, I_2, I_3, I_4$, with closed-form Riccati solutions for $I_3$ and $I_4$:

$$I_3=\frac{A_1-(2j\delta\lambda\rho_2\phi-2\bar{\alpha})}{4\delta^2}\cdot\frac{1-e^{A_1\tau}}{1-B_1e^{A_1\tau}}, \tag{27}$$

$$I_4=\frac{A_2-(j\sigma\rho_1\phi-\bar{\eta})}{\sigma^2}\cdot\frac{1-e^{A_2\tau}}{1-A_2e^{A_2\tau}}. \tag{28}$$

Given $I_3$, we solve:

$$I_2=-\frac{\bar{\alpha}\bar{\beta}}{\delta^2}\left(A_1-2(j\delta\lambda\rho_2\phi-\bar{\alpha})\right)\frac{(1-e^{\frac{1}{2}A_1\tau})^2}{1-B_1e^{A_1\tau}}. \tag{29}$$

Finally, $I_1$ is integrated explicitly:

$$I_1=\tfrac{1}{4}\{(A_1-2(j\delta\lambda\rho_2\phi-\bar{\alpha}))\,\tau-2\ln\left(\tfrac{1-B_1e^{A_1\tau}}{1-B_1}\right)\}$$

$$+\frac{\overline{\eta\theta}}{\sigma^2}\{[A_2-(j\sigma\rho_1\varphi-\overline{\eta})]\tau-2\ln\left(\frac{1-B_2e^{A_2\tau}}{1-B_2}\right)\}+\int_0^\tau\left(\frac{1}{2}\delta^2I_2^2(\varphi;s)+\overline{\alpha}\,\overline{\beta}I_2(\varphi;s)\right)ds+jr\tau\varphi. \tag{30}$$

# 4 Numerical Validation and Sensitivity Analysis

To validate the analytical pricing formula, we first benchmark it against Monte Carlo simulations. This comparison confirms the accuracy of the formula and enables further investigation

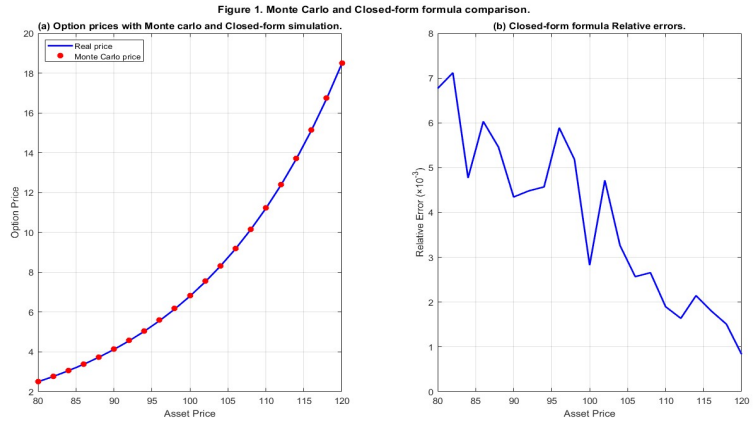



Figure 1: Write the caption here.

into the effects of volatility and liquidity risks. Unless otherwise specified, the following parameters are used for numerical evaluation:

$$\bar{\eta}=7,\ \bar{\theta}=0.06,\ \sigma=0.17,\ \bar{\alpha}=3,\ \bar{\beta}=0.3\ \delta=0.1,\ \lambda=0.6,\ \rho_1=-0.6,$$

$$\rho_2=-0.3,\ r=0.02,\ \tau=1,\ P_0=100,\ K=100,\ M_0=0.1,\ v_0=0.03.$$

Figure 1 demonstrates strong agreement between the option prices computed via the closed-form formula and those obtained from simulations. In particular, Figure 1 (a) shows pointwise consistency, and Figure 1 (b) confirms a maximum relative error below 0.6%, establishing the formula's precision. Next, we examine how the liquidity sensitivity parameter $\lambda$ influences option prices. Three model variations are compared:

- **CV Model**: Ignores stochastic volatility ($\bar{\eta} = \sigma = 0$); based on Feng and *et al.* (2014).
- **CL Model**: Omits stochastic liquidity ($\bar{\alpha} = \delta = 0$).
- **Full Model**: Incorporates both stochastic volatility and liquidity.

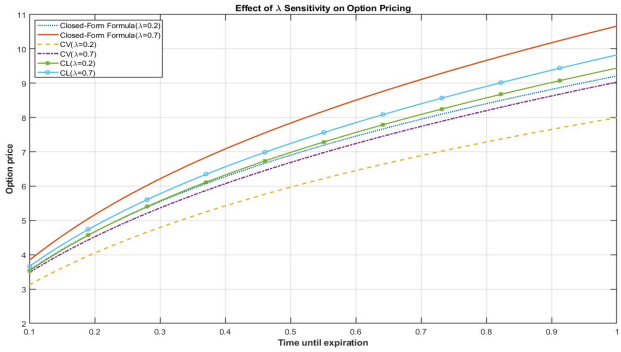



Figure 2: Write the caption here.

Results in Figure 2 show that increasing $\lambda$ leads to higher option prices across all models. This is, expected as greater sensitivity to market liquidity implies a stronger discounting effect, raising option value. Moreover, both stochastic volatility and stochastic liquidity are shown to have a significant impact on pricing, motivating a deeper analysis of their combined effects. This gives a fully analytical form for the characteristic function, enabling efficient computation of the European option price under both volatility and liquidity risks.

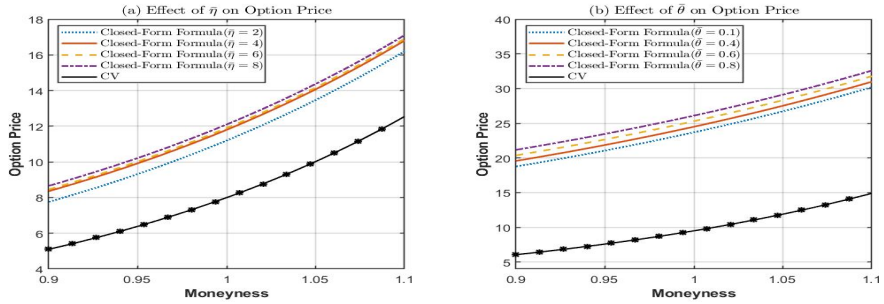



Figure 3: Write the caption here.

To evaluate the role of stochastic volatility, we benchmark against the CV model (Feng and *et al.* , 2014). Figure 3 (a) shows that option prices increase with the mean reversion speed $\bar{\eta}$, due to faster convergence toward a higher long-term volatility level $\bar{\theta}$. However, this effect tapers off as $\bar{\eta}$ becomes large. Conversely, if $\bar{\theta}$ were below the current variance, the trend would reverse. Figure 3 (b) confirms that raising $\bar{\theta}$ also elevates option values by amplifying expected volatility. Across both cases, the CV model yields the lowest prices, given its constant and relatively low volatility assumption, though this could change with different parameter settings. Figure 4 (a) illustrates how option prices respond to changes in the volatility of volatility parameter $\sigma$. A noteworthy pattern emerges: when the option is deep out-of-the-money (low moneyness), increasing $\sigma$ tends to reduce its value. In contrast, for deep in-the-money options (high moneyness), greater $\sigma$ leads to higher option prices. This asymmetry can be explained by exercise probability: high volatility decreases the chance of realization for out-of-the-money options while potentially amplifying gains for in-the-money options. To assess the role of stochastic liquidity, we compare our full model with the CL model under varying liquidity parameters. Figure 4 (b) shows that option prices rise with the

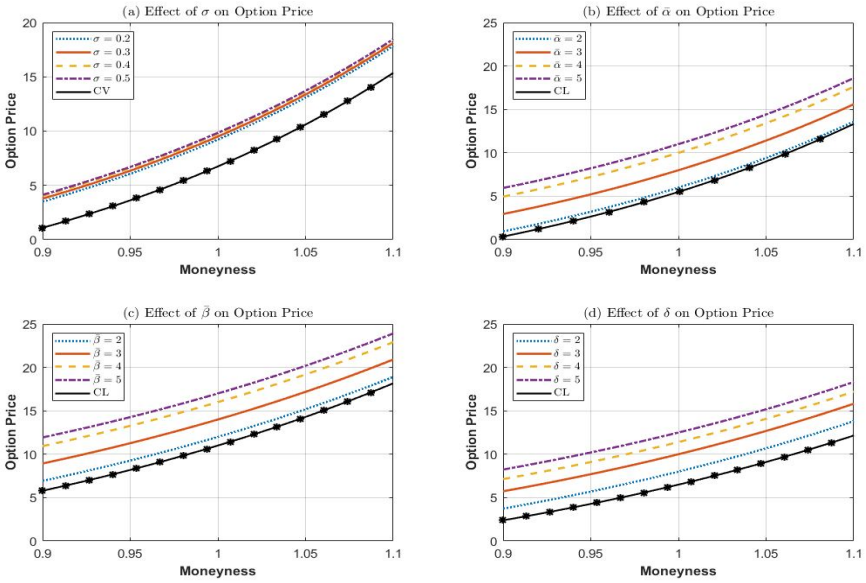



Figure 4: Write the caption here.

mean-reversion speed $\bar{\alpha}$, since faster convergence to a higher long-term illiquidity level increases perceived liquidity risk, thereby raising the option premium. Figure 4 (c) examines the impact of the equilibrium liquidity level $\bar{\beta}$, where a clear upward trend in option prices is observed. Higher $\bar{\beta}$ reflects more persistent illiquidity, leading to greater pricing impact. The CL model, which assumes a lower fixed liquidity level, consistently yields lower option prices. This pattern could reverse if the CL model were set with a higher illiquidity baseline. Finally, Figure 4 (d) evaluates how volatility in market liquidity affects pricing. As liquidity volatility rises, option prices increase. This is intuitive: higher uncertainty in liquidity conditions discourages option sellers from offering contracts at lower prices due to elevated exposure to liquidity shocks.

# Discussion and Results

In response to the demand for more realistic financial market modeling, this paper introduces a novel option pricing framework that incorporates both stochastic volatility and liquidity risks. The model extends the classical Heston stochastic volatility framework by introducing a liquidity discounting mechanism that captures stochastic market liquidity conditions. A risk-neutral measure is constructed via benchmark transformation to facilitate derivative pricing. Within this framework, we derive a closed-form solution for European option prices. Numerical experiments validate the model's effectiveness and demonstrate its capability to reflect key empirical characteristics of financial markets, particularly those related to volatility and liquidity dynamics.

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