Diagnostic methods Using Liu Estimator in Linear Mixed Measurement Error Models

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Abstract: In this paper, we focused on identifying influential observations using Liu corrected likelihood estimator (LCLE) in linear mixed measurement error models when mul-ticollinearity is present. We considered an extension of Cook's distance to determine influential observations based on the case deletion model. Finally, simulation study was provided to illustrate the performance of the influence measures.

Keywords: Diagnostic · Influential observations -Cook's distance -Liu estimator

1 Introduction

Consider the following linear mixed model with measurement errors

$$\boldsymbol{y} = \boldsymbol{Z}\boldsymbol{\beta} + \boldsymbol{U}\boldsymbol{b} + \boldsymbol{\varepsilon}, \qquad \boldsymbol{X} = \boldsymbol{Z} + \Delta, \tag{1}$$

Where $\Delta' = [\delta_1, \delta_2, ..., \delta_n]$ and $\boldsymbol{b} \sim N(\boldsymbol{0}, \sigma^2\boldsymbol{\Sigma})$ that $\boldsymbol{\Sigma}$ is a block diagonal matrix with the *i*th block being $\gamma_i \boldsymbol{I}_{q_i}$ for $\gamma_i = \frac{\sigma_i^2}{\sigma^2}$, so that $\boldsymbol{y}$ has a multivariate normal distribution with $E(\boldsymbol{y}) = \boldsymbol{Z}\boldsymbol{\beta}$ and $Var(\boldsymbol{y}) = \sigma^2\boldsymbol{V}$, in which $\boldsymbol{V} = \boldsymbol{I}_n + \sum_{i=1}^{m} \gamma_i \boldsymbol{U}_i \boldsymbol{U}_i' = \boldsymbol{I}_n + \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{U}'$. Then the corrected likelihood estimates (CLE) of estimates are given by $\hat{\boldsymbol{\beta}} = \left[\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X} - tr(\boldsymbol{V}^{-1})\boldsymbol{\Lambda}\right]^{-1}\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{y} = \boldsymbol{A}^{-1}\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{y}$ where $\boldsymbol{A} = \left[\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X} - tr(\boldsymbol{V}^{-1})\boldsymbol{\Lambda}\right]$, $\hat{\sigma}^2 = n^{-1}\left[(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}})'\boldsymbol{V}^{-1}(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}) - tr(\boldsymbol{V}^{-1})\hat{\boldsymbol{\beta}}'\boldsymbol{\Lambda}\hat{\boldsymbol{\beta}}\right]$ and $\hat{\sigma}_i^2 = \frac{\tilde{\boldsymbol{b}}_i'\tilde{\boldsymbol{b}}_i - tr(\hat{\boldsymbol{D}}_i'\hat{\boldsymbol{D}}_i)\hat{\boldsymbol{\beta}}'\boldsymbol{\Lambda}\hat{\boldsymbol{\beta}}}{[q_i - tr(T_{ii})]}; i = 1, ..., m$ in which $\hat{\boldsymbol{D}}_i = \hat{\gamma}_i \boldsymbol{U}_i'\boldsymbol{V}^{-1} = \frac{\hat{\sigma}_i^2}{\hat{\sigma}^2}\boldsymbol{U}_i'\boldsymbol{V}^{-1}$ and $\tilde{\boldsymbol{b}} = \boldsymbol{\Sigma}\boldsymbol{U}'\boldsymbol{V}^{-1}(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}})$ is random effects prediction and $\boldsymbol{T}_{ij}$ is *ij*th block of matrix $\boldsymbol{T}$ (see Zhong et al., 2002 and Zare et al., 2012).

Linear mixed models have tremendous flexibility to fit statistical models and are used to analyze data over a wide range of topics .It is a very common problem to deal with highly intercorrelated explanatory variables. There are several ways to overcome this problem such as Stein estimator (Stein, 1956) and Liu estimator (Liu, 1993). Sensitivity and residual analysis of the underlying assumptions constitute important tools for evaluating the fit of any statistical model to given data.

For this end, based on Liu (1993) we consider the linear restriction $d\hat{\boldsymbol{\beta}} = \boldsymbol{\beta} + \boldsymbol{e},\ \boldsymbol{e} \sim N(\mathbf{0}, \sigma^2 \boldsymbol{I}_p)$ where $0 < d < 1$ is the Liu biasing parameter and $\hat{\boldsymbol{\beta}}$ is the corrected likelihood estimator (CLE) in linear mixed measurement error model. The LCLE of $\boldsymbol{\beta}$ and σ^2 will be obtained as (Ghapani, 2020). $\hat{\boldsymbol{\beta}}_d = \boldsymbol{A}_p^{-1}(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{y} + d\hat{\boldsymbol{\beta}})$ and

$$\hat{\sigma}_d^2 = n^{-1}\left[(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}_d)'\boldsymbol{V}^{-1}(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}_d) - tr(\boldsymbol{V}^{-1})\hat{\boldsymbol{\beta}}_d'\boldsymbol{\Lambda}\hat{\boldsymbol{\beta}}_d + (d\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_d)'(d\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_d)\right]$$

where $\boldsymbol{A}_p = (\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X} - tr(\boldsymbol{V}^{-1})\boldsymbol{\Lambda} + \boldsymbol{I}_p)$. Also, the Liu predictor of $\boldsymbol{b}$ is given by

$$\tilde{\boldsymbol{b}}_d = \Sigma \boldsymbol{U}'\boldsymbol{V}^{-1}(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}_d).$$

Theorem 1: $\hat{\boldsymbol{\beta}}_d$ has an asymptotically normal distribution with mean $AE(\hat{\boldsymbol{\beta}}_d) = \boldsymbol{G}_d\boldsymbol{\beta}$ and covariance matrix $AVar(\hat{\boldsymbol{\beta}}_d) = \boldsymbol{G}_d(\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z})^{-1}(\boldsymbol{B} + \sigma^2\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z})(\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z})^{-1}\boldsymbol{G}_d$, which $\boldsymbol{B} = n\sigma^2 + \boldsymbol{\beta}'\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z}\boldsymbol{\beta}$ and $\boldsymbol{G}_d = (\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z} + \boldsymbol{I}_p)^{-1}(\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z} + d\boldsymbol{I}_p)$. (Proof: see Ghapani, 2020)

Corollary: $\hat{\boldsymbol{\beta}}$ has an asymptotically normal distribution with mean $AE(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}$ and covariance matrix $AVar(\hat{\boldsymbol{\beta}}) = (\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z})^{-1}(\boldsymbol{B} + \sigma^2\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z})(\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z})^{-1}$.

Theorem 2: There exists $0 < l_i = \dfrac{\lambda_i\alpha_i^2 - \sigma^2(2\lambda_i + 1)}{\lambda_i\alpha_i^2 + \sigma^2} < d < 1,\ i = 1, ..., p$ such that $\boldsymbol{\Delta}$ is a positive definite matrix. (explanation : The spectral decomposition of the matrix $\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z} > 0$ is $\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}'$ where $\boldsymbol{\Lambda} = diag(\lambda_1, ..., \lambda_p)$, $\lambda_i > 0$ and λ_i are eigenvalues of $\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z}$, there exists some orthogonal matrix $\boldsymbol{Q}$, We denote $\boldsymbol{\alpha} = \boldsymbol{Q}'\boldsymbol{\beta}$, then, we have $\boldsymbol{\beta} = \boldsymbol{Q}\boldsymbol{\alpha}$.) (Proof: see Ghapani, 2020)

3. Diagnostic methods

To determine the observations that behave abnormally compared to other observations, various methods have used by statisticians. We study some of these measures in the context of linear mixed measurement error models under the LCLE.

3.1. Case deletion diagnostic

To quantify the effects of deleting the *i*th observation on estimation of parameters, a fundamental approach is case deletion model that, for the Liu estimator with the *i*th observation, deleted in linear mixed measurement error models, is defined as

$$\begin{aligned} &\boldsymbol{y}_{(i)} = \boldsymbol{Z}_{(i)}\boldsymbol{\beta} + \boldsymbol{U}_{(i)}\boldsymbol{b} + \boldsymbol{\varepsilon}_{(i)},\ \boldsymbol{X}_{(i)} = \boldsymbol{Z}_{(i)} + \Delta_{(i)},\ i = 1, 2, \ldots n \\ &d\hat{\boldsymbol{\beta}} = \boldsymbol{\beta} + \boldsymbol{e}, \end{aligned} \tag{2}$$

Theorem 4: For model (2), we have

$$\hat{\boldsymbol{\beta}}_{d(i)} \simeq \hat{\boldsymbol{\beta}}_d - \boldsymbol{A}_p^{-1}\frac{\boldsymbol{X}'\boldsymbol{c}_i\hat{\boldsymbol{v}}_{di}}{r_{pii}}$$

$$\hat{\sigma}^2_{d(i)} \simeq \frac{n}{n-1}\hat{\sigma}^2_d - \boldsymbol{c}_i'(y - \boldsymbol{X}\hat{\boldsymbol{\beta}}_d)\frac{\hat{\boldsymbol{v}}_{di}}{(n-1)r_{pii}} = \frac{n}{n-1}\hat{\sigma}^2_d - \frac{\hat{\boldsymbol{v}}^2_{di}}{(n-1)r_{pii}}$$

$$\tilde{\boldsymbol{b}}_{d(i)} \simeq \tilde{\boldsymbol{b}}_d - \boldsymbol{\Sigma}\boldsymbol{U}'\boldsymbol{r}_{pi}\frac{\hat{\boldsymbol{v}}_{di}}{r_{pii}}$$

where $\hat{\boldsymbol{\beta}}_{d(i)}$, $\hat{\sigma}^2_{d(i)}$ and $\tilde{\boldsymbol{b}}_{d(i)}$ denote the estimates of $\boldsymbol{\beta}$, σ^2 and $\boldsymbol{b}$ when the *i*th case is deleted, respectively. Also, $\boldsymbol{c}_i'$ and $\boldsymbol{r}_{pi}'$ are *i*th rows of $\boldsymbol{V}^{-1}$ and $\boldsymbol{R}_p = \boldsymbol{V}^{-1} - \boldsymbol{V}^{-1}\boldsymbol{X}\boldsymbol{A}_p^{-1}\boldsymbol{X}'\boldsymbol{V}^{-1}$, respectively, c_{ii} and r_{pii} denotes the *i*th diagonal elements of $\boldsymbol{V}^{-1}$ and $\boldsymbol{R}_p$, $\hat{\boldsymbol{v}}_{di} = \boldsymbol{c}_i'(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}_d)$ is the *i*th elements of $\hat{\boldsymbol{v}}_d$.

Proof: When the *i*th row is deleted from the data set, we have

$$\hat{\boldsymbol{\beta}}_{d(i)} = (\boldsymbol{X}_{(i)}'\boldsymbol{V}_{[i]}^{-1}\boldsymbol{X}_{(i)} - tr(\boldsymbol{V}_{[i]}^{-1})\boldsymbol{\Lambda} + \boldsymbol{I}_p)^{-1}(\boldsymbol{X}_{(i)}'\boldsymbol{V}_{[i]}^{-1}\boldsymbol{y}_{(i)} + d\hat{\boldsymbol{\beta}}),$$

where $\boldsymbol{V}_{[i]}^{-1}$ denotes inverse matrix $\boldsymbol{V}$ with the *i*th row and column removed. Using $(\boldsymbol{A} + \boldsymbol{BCD})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{C}^{-1} + \boldsymbol{D}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1}\boldsymbol{D}\boldsymbol{A}^{-1}$ (Rao et al. 2008, Theorem A. 18), $\hat{\boldsymbol{\beta}}_{d(i)}$ can be written as

$$\hat{\boldsymbol{\beta}}_{d(i)} = (\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X} - \frac{\boldsymbol{X}'\boldsymbol{c}_i\boldsymbol{c}_i'\boldsymbol{X}}{c_{ii}} - tr(\boldsymbol{V}^{-1})\boldsymbol{\Lambda} + c_{ii}\boldsymbol{\Lambda} + \boldsymbol{I}_p)^{-1}(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{y} - \frac{\boldsymbol{X}'\boldsymbol{c}_i\boldsymbol{c}_i'\boldsymbol{y}}{c_{ii}} + d\hat{\boldsymbol{\beta}})$$

$$= (\boldsymbol{A}_p^{-1} + \boldsymbol{A}_p^{-1}\frac{\boldsymbol{X}'\boldsymbol{c}_i}{c_{ii}}(\boldsymbol{I}_n - \frac{\boldsymbol{c}_i'\boldsymbol{X}\boldsymbol{A}_p^{-1}\boldsymbol{X}'\boldsymbol{c}_i}{c_{ii}})^{-1}\boldsymbol{c}_i'\boldsymbol{X}\boldsymbol{A}_p^{-1})(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{y} + d\hat{\boldsymbol{\beta}} - \frac{\boldsymbol{X}'\boldsymbol{c}_i\boldsymbol{c}_i'\boldsymbol{y}}{c_{ii}}) + \boldsymbol{O}_p(n^{-1})$$

$$\simeq \hat{\boldsymbol{\beta}}_d - \boldsymbol{A}_p^{-1}\frac{\boldsymbol{X}'\boldsymbol{c}_i\hat{\boldsymbol{v}}_{di}}{r_{pii}}.$$

As the same way

$$(n-1)\hat{\sigma}^2_{d(i)} = (\boldsymbol{y}_{(i)} - \boldsymbol{X}_{(i)}\hat{\boldsymbol{\beta}}_{d(i)})'\boldsymbol{V}_{[i]}^{-1}(\boldsymbol{y}_{(i)} - \boldsymbol{X}_{(i)}\hat{\boldsymbol{\beta}}_{d(i)}) - tr(\boldsymbol{V}_{[i]}^{-1})\hat{\boldsymbol{\beta}}_{d(i)}\boldsymbol{\Lambda}\hat{\boldsymbol{\beta}}_{d(i)}$$
$$-(d\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_{d(i)})'(d\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_{d(i)})$$

Then, $\hat{\sigma}^2_{d(i)} \simeq \frac{n}{n-1}\hat{\sigma}^2_d - \boldsymbol{c}_i'(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}_d)\frac{\hat{\boldsymbol{v}}_{di}}{(n-1)r_{pii}} = \frac{n}{n-1}\hat{\sigma}^2_d - \frac{\hat{\boldsymbol{v}}^2_{di}}{(n-1)r_{pii}}$. (4)

$$\tilde{\boldsymbol{b}}_{d(i)} = \boldsymbol{\Sigma}\boldsymbol{U}_{(i)}'\boldsymbol{V}_{[i]}^{-1}(\boldsymbol{y}_{(i)} - \boldsymbol{X}_{(i)}\hat{\boldsymbol{\beta}}_{d(i)}) \simeq \tilde{\boldsymbol{b}}_d - \boldsymbol{\Sigma}\boldsymbol{U}'(\boldsymbol{c}_i - \boldsymbol{V}^{-1}\boldsymbol{X}\boldsymbol{A}_p^{-1}\boldsymbol{X}'\boldsymbol{c}_i)\frac{\hat{\boldsymbol{v}}_{di}}{r_{pii}}$$

And

$$= \tilde{\boldsymbol{b}}_d - \boldsymbol{\Sigma}\boldsymbol{U}'\boldsymbol{r}_{pi}\frac{\hat{\boldsymbol{v}}_{di}}{r_{pii}}$$

3.2. Generalized Cook's distance

Cook's distance is a convenient measure of influence, which based on Cook (1977) is denoted by under LCLE is given as linear mixed measurement error modelsand in $CD_i(\boldsymbol{\beta})$

$$CD_i(\boldsymbol{\beta}) = (\hat{\boldsymbol{\beta}}_d - \hat{\boldsymbol{\beta}}_{d(i)})'\boldsymbol{M}(\hat{\boldsymbol{\beta}}_d - \hat{\boldsymbol{\beta}}_{d(i)})',$$

where $\boldsymbol{M} = \hat{\sigma}_d^{-2}\left[\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X} - tr(\boldsymbol{V}^{-1})\boldsymbol{\Lambda} + \boldsymbol{I}_p\right]$. Then, we have

$$CD_i(\boldsymbol{\beta}) = \frac{(\hat{\boldsymbol{\beta}}_d - \hat{\boldsymbol{\beta}}_{d(i)})'\left[\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X} - tr(\boldsymbol{V}^{-1})\boldsymbol{\Lambda} + \boldsymbol{I}_p\right](\hat{\boldsymbol{\beta}}_d - \hat{\boldsymbol{\beta}}_{d(i)})}{\hat{\sigma}_d^2},$$

then we have approximately $CD_i(\boldsymbol{\beta}) = \frac{(c_{ii} - r_{pii})\hat{\boldsymbol{v}}^2_{di}}{\hat{\sigma}_d^2 r_{pii}^2} + \boldsymbol{O}_p(n^{-1})$.

3.3. Cook's distance for random effects

The Cook's distance for random effects is define as

$$CD_i(\boldsymbol{b}) = \frac{(\tilde{\boldsymbol{b}}_d - \tilde{\boldsymbol{b}}_{d(i)})'(\boldsymbol{U}'\boldsymbol{U} + \boldsymbol{\Sigma}^{-1})(\tilde{\boldsymbol{b}}_d - \tilde{\boldsymbol{b}}_{d(i)})}{\hat{\sigma}_d^2}.$$

Since $\tilde{\boldsymbol{b}}_{d(i)} \simeq \tilde{\boldsymbol{b}}_d - \Sigma \boldsymbol{U}'\boldsymbol{r}_{pi} \frac{\hat{\boldsymbol{v}}_{di}}{r_{pii}}$, we can get $CD_i(\boldsymbol{b}) = \boldsymbol{r}'_{pi}(\boldsymbol{V} - \boldsymbol{I}_n)\boldsymbol{V}\boldsymbol{r}_{pi} \frac{\hat{\boldsymbol{v}}_{di}^2}{\hat{\sigma}_d^2 r_{pii}^2}$.

Simulation study

To assess the performance of the $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$ in terms of type I error and power of test, we carried out the following parametric bootstrap simulation study. The *i*th simulated data set was generated as

$$\begin{aligned} \boldsymbol{y}_i &= \boldsymbol{Z}\boldsymbol{\beta} + \boldsymbol{U}\boldsymbol{b}_i + \boldsymbol{\varepsilon}_i, \\ \boldsymbol{X}_i &= \boldsymbol{Z} + \boldsymbol{\Delta} \end{aligned} \qquad i = 1,...,1000 \qquad (5)$$

Where $\boldsymbol{y}_i = (y_{11i},...,y_{1qi},y_{21i},...,y_{2qi},...,y_{m1i},...,y_{mqi})'$, $\boldsymbol{b}_i = (b_{1i},b_{1i},...,b_{1i})'$, $Z = (\mathbf{z}^{(1)},\mathbf{z}^{(2)},\mathbf{z}^{(3)})$, $\mathbf{z}^{(s)} = (z_{11}^{(s)},...,z_{1q}^{(s)},z_{21}^{(s)},...,z_{2q}^{(s)},...,z_{m1}^{(s)},...,z_{mq}^{(s)})', s = 1,2,3$, and $\boldsymbol{\varepsilon}_i$ is rewritten in accordance with $\boldsymbol{y}_i$. $U = \mathbf{1}_q \oplus \mathbf{1}_q \oplus ... \oplus \mathbf{1}_q$ is an $n \times m$ matrix, where $\mathbf{1}_q$ is a $q \times 1$ vector all of whose elements are 1's (see, Searle, 1982, p. 264). Furthermore, *m* can be interpreted as the number of independent groups; q is the group size and $n = mq$ is the total size (Wang *et al.*,1998). We consider the following combinations for simulation: $m = 7, q = 4,10$ (and so n= 28,70), $\boldsymbol{\beta} = (\beta_1,\beta_2,\beta_3) = (3,1,2)'$, $z^{(1)} \sim N(3,1), z^{(2)} \sim N(3,1), z^{(3)} = z^{(3)} + \upsilon, \upsilon \sim N(0,1)$ for i $= 1,...,n$ and $\boldsymbol{b}_i \sim N(0,\sigma_1^2)$ for $\sigma_1^2 = 0.16$ and 0.25 and the random errors $\varepsilon_i \sim N(0,\sigma^2)$ for $\sigma^2 = 0.36$ and 0.64 and $\boldsymbol{\Lambda} = diag(0,0.05,0.05,0.05)$. Furthermore, for each simulated data set, $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$ were calculated for the first observation. The choice of the first observation was arbitrary. To generate an empirical distribution (see Ghapani 2019) of the test statistic under the null hypothesis, the data sets for $j = 1,...,1000$ were simulated as

$$\begin{aligned} \boldsymbol{y}_{ij}^* &= \hat{\boldsymbol{Z}}_{dj}\hat{\boldsymbol{\beta}}_{dj} + \boldsymbol{U}\boldsymbol{b}_{ij}^* + \boldsymbol{\varepsilon}_{ij}^*, \\ \boldsymbol{X}_{ij}^* &= \hat{\boldsymbol{Z}}_{dj} + \boldsymbol{\Delta}_{ij} \end{aligned}$$

where $\hat{\boldsymbol{\beta}}_{dj}$, $\hat{\boldsymbol{Z}}_{dj}$, $\hat{\sigma}_i^2$ and $\hat{\sigma}_{1i}^2$ are the are the corrected Liu estimates of $\boldsymbol{\beta}$, $\boldsymbol{Z}$, σ^2 and σ_1^2 from model (7). Also $\boldsymbol{\varepsilon}_{ij}^*$, $\boldsymbol{b}_{ij}^*$ and $\boldsymbol{\Delta}_{ij}$ have a normal distribution with zero mean and variances $\hat{\sigma}_i^2\mathbf{I}_n$, $\hat{\sigma}_{1i}^2\mathbf{I}_q$ and $\mathbf{I}_n \otimes \boldsymbol{\Lambda}$ respectively. The $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$ were performed for the first observation of each simulated data and $100(1-\alpha)$ percentiles from the empirical distribution of test statistic was used as threshold value of the test statistic of model (5). The probability of a type I error estimate for the $CD_i(\boldsymbol{\beta}_d)$, $CD_i(\boldsymbol{b}_d)$ and $\alpha = 0.05$ were calculated as the number of data sets for which the $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$ exceeded the $100(1-\alpha)$ percentiles of the empirical distribution, divided by the number of replicates. It appears that in general the type I error of the $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$ for different combinations of parameters are close to the nominal value of 0:05. In the next step, in order to evaluate the relative sensitivity of the $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$, we introduce $z^{(1)} \sim N(5,3), z^{(2)} \sim N(5,3), z^{(2)} = z^{(3)} + \upsilon,\ \upsilon \sim N(0,1)$

generated as a point with high leverage for the first observation and again for each combination of parameters, 1000 data sets are generated from the following model

$$\begin{aligned} \boldsymbol{y}_i &= \boldsymbol{Z}\boldsymbol{\beta} + \boldsymbol{U}\boldsymbol{b}_i + \boldsymbol{\varepsilon}_i, \\ \boldsymbol{X}_i &= \boldsymbol{Z} + \boldsymbol{\Delta} \end{aligned} \quad i = 1,...,1000$$

he parameters remained as in the evaluation of type I error. Again, for each simulated data set, we derive the Liu estimate of parameters and the $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$ for the first observation. The power of the $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$ were calculated as the number of data sets for which the $CD_i(\boldsymbol{\beta}_d)$ and $CD_i(\boldsymbol{b}_d)$ exceeded the percentile $100(1-\alpha)$ of the empirical distribution, divided by the number of replicates. The results are presented in Table 1. According to Table 1, we found that the estimates obtained for the linear mixed measurement error models under LCLE are more powerful than the model mixed with the measurement error.

Table1: Comparison of the power of Cook's distance for CLE and LCLE

n	(σ_1^2, σ^2)	$CD(\boldsymbol{\beta})$		$CD(\boldsymbol{\beta}_d)$		$CD(\boldsymbol{b})$		$CD(\boldsymbol{b}_d)$	
		Sig level	Power	Sig level	Power	Sig level	Power	Sig level	Power

70	(0.16,0.36)	0.048	0.634	0.031	0.676	0.047	0.113	0.043	0.123
70	(0.25,0.64)	0.040	0.599	0.030	0.635	0.036	0.083	0.044	0.118
28	(0.16,0.36)	0.064	0.422	0.04	0.456	0.063	0.101	0.040	0.129
28	(0.25,0.64)	0.067	0.418	0.039	0.450	0.070	0.048	0.036	0.094

Discussion

In this paper, we have presented diagnostic methods based on LCLE in linear mixed measurement error models. Based on LCLE, we obtained case deletion diagnostics for the detection of the influential observations in these models. The final numerical results and simulation studies have demonstrated that the diagnostic methods presented in this paper are useful to detect influential observations.

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