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Determination of Minimal Weighted Risk Sample Sizes in Sampling Inspections with Stochastic defect rate

M. Naghizadeh Qomi*

Department of Statistics, University of Mazandaran, Babolsar, Iran

Abstract

Single reliability inspection of manufactured lots is considered to determine optimal sample size using empirical data and prior information. A Beta model is assumed to describe the stochastic behavior of the proportion nonconforming or the defect rate, p. Optimal test plans are derived for the Tsallis q-exponential lifetime distributions by minimizing the classical and expected weighted-average of producer and consumer risks. Integer nonlinear programming is used to obtain optimal sample sizes. The results show that using prior information for defective rates reduces the sample size in the selected sample. A real data example is applied to illustrate the proposed plan.

Keywords: Acceptance sampling inspection plans; Beta prior model; integer nonlinear programming; weighted average risks; Tsallis q-exponential distributions.
Mathematics Subject Classification (2020): 62N05, 62N10.

1 Introduction

One of the challenges for manufacturers and customers in the market is the quality of manufactured products. The manufacturer seeks to produce a quality product that is acceptable to the customer, and the customer seeks to receive quality products. Acceptance sampling plans are one of the important tools in quality control. Therefore, there is a need to inspect production batches. Since full batch inspection is not possible due to time and cost constraints, acceptance sampling plans *ASPs* are used as one of the important quality control tools. *ASPs* have been studied by many authors, the most recent of which are Fernández (2022), Pérez-González et al. (2023), Naghizadeh Qomi and Fernández (2024), Tripathi et al. (2024) and Naghizadeh Qomi and Tripathi (2025).

Single acceptance sampling inspection plan (*SASIP*) is a conventional approach for determining the acceptability of submitted lots. In this plan, a random sample of units are selected from the submitted lot and the lot will be accepted if the number of defective units are no more than an acceptance number. This type of *ASPs* is developed by several researches,

*Corresponding author, m.naghizadeh@umz.ac.ir

among others we can cite Al-Nasser and Obeidat (2020), Tripathi et al. (2022), Tripathi et al. (2023), Al-Husseini et al. (2023) and Saha et al. (2024) which discussed single sampling plans for Tsallis q-exponential, generalized half-normal, transmuted Rayleigh, exponentiated moment exponential and logistic Rayleigh distributions, respectively.

Al-Nasser and Obeidat (2020) have recently conducted a time-censored *SASIP* using single point method (limiting consumer risk) when the lifetime distribution follows the Tsallis q-exponential (TQE) distribution. Our motivation is to a construct a time-censored *SASIP* by minimizing the weighted-average risks in order to decide the acceptability of a submitted lot of products using empirical data and prior information.

The remainder of the paper is organized as follows. The *TQE* distribution is briefly introduced in Section 2. The methodology of *SASP* is proposed in Section 3. The *SASP*s with minimal weighted-average risks in both frequentist and Bayesian framework are presented in Sections 4 and 5, respectively. A real data example is provided in Section 6. Finally, a brief discussion is presented in Section 7.

2 Tsallis q-exponential distributions

The TQE model was derived by maximizing the Tsallis entropy

$$S_q(f) = \frac{1}{q-1}\left[1 - \int_{-\infty}^{\infty} \{f(x)\}^q \, \mathrm{d}x\right],$$

where $q \neq 1$ is a real parameter, sometimes called entropic-index, and $f(x)$ is a probability density function (pdf). The TQE distribution converges to the ordinary exponential distribution with parameter λ when $q \to 1$. The TQE distribution is a generalization of some lifetime distributions such as the Lomax model, and it is a particular case of the generalized type II Pareto distribution; see Al-Nasser and Obeidat (2020).

According to Picoli et al. Picoli et al. (2009), the TQE model would be a good representation of both short and long-tailed distributions by choosing a suitable value of q, and therefore could be considered advantageous over many other distributions in the area of acceptance sampling. The pdf of the TQE distribution is given by

$$f(t; q, \theta) = (2-q)\theta[1 + (q-1)\theta t]^{1/(1-q)}, \quad t > 0, \tag{1}$$

and the corresponding cumulative distribution function (cdf) is as

$$F(t; q, \theta) = 1 - [1 + (q-1)\theta t]^{(2-q)/(1-q)}, \quad t > 0, \tag{2}$$

where $q < 2$ ($q \neq 1$) and $\theta > 0$ are the shape and scale parameters, respectively. We denote the random variable T with density (1) by writting $T \sim TQE(q, \theta)$. The mean of the TQE distribution is given by

$$\mu = \frac{1}{\theta(3-2q)}, \quad q < \frac{3}{2}. \tag{3}$$

which is inverse proportional to the scale parameter θ when the parameter q is fixed. For simplicity, the mean lifetime is considered the quality characteristic of interest. However, note that in our context it is equivalent to use the mean or median lifetimes, or even a given percentile.

3 Design of time-censoring *SASIPs*

Consider that a large lot of products is submitted to judge its acceptability and the lifespan of products follows the $TQE(q, \theta)$ with pdf given in (1). Moreover, assume that p denotes the true proportion nonconforming which would approximately be the fraction of defective objects in the lot. Also, let μ be the true mean lifetime of products and μ_0 be the specified mean life.

The quality level of a product can be expressed in terms of the ratio $r = \mu/\mu_0$. A lot is said to be good if $r \geq 1$ and is considered bad if $r < 1$. The classical producer and consumer risks are defined respectively as probabilities of good lot rejection and bad lot acceptance.

The decision about lot is based on the number X_n of defective items where X_n assumed to be binomial with parameters $n \in \mathcal{N} = \{1, 2, 3, ...\}$ and $p \in (0, 1)$. Given the fixed test time $t > 0$, an *SASIP* is characterized by $S_d \equiv (n, c, f)$, where $f = t/\mu_0$ is test termination. The decision process according to the inspection plan S_d can be stated as follows:

- *Step 1:* Select n items randomly from the submitted lot.

- *Step 2:* Accept the lot if the number X_n of defective items are no more than a value c known as acceptance number.

The operating characteristic (OC) function describes the stochastic behavior of the test plan S_d and is defined as the lot acceptance probability. Hence, the OC function can be expressed by

$$\begin{aligned} \mathcal{L}(p) &\equiv \mathcal{L}(p, n, c) \\ &= P(X_n \leq c|p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}. \end{aligned} \tag{4}$$

From (2) and (3), the proportion defective before time t is $p \equiv p(f, q, r)$ of the form

$$p = 1 - \left[1 + \frac{(q-1)f}{r(3-2q)}\right]^{(2-q)/(1-q)}.$$

4 *SASIPs* with minimal classical weighted-average risks

Let r_0 and r_1 be the ratio of quality level corresponding to the producer and consumer, respectively. Moreover, $p_0 \equiv p(d, f, r_0)$ and $p_1 \equiv p(d, f, r_1)$ are the acceptable and rejectable defective rates associated with the producer and consumer, respectively. The classical producer and consumer risks (CPR and CCR) are defined respectively as

$$CPR(n, c, p_0) = \sum_{i=c+1}^{n} \binom{n}{i} p_0^i (1-p_0)^{n-i},$$

and

$$CCR(n, c, p_1) = \sum_{i=0}^{c} \binom{n}{i} p_1^i (1-p_1)^{n-i}.$$

Following Fernández et al. (2020), consider the classical weighted-average risk (CWR) as

$$CWR(n, c, p_0, p_1) = w_0 CPR(n, c, p_0) + w_1 CCR(n, c, p_1), \quad w_0, w_1 > 0, \tag{5}$$

where w_0 and w_1 are the producer and consumer weights, respectively and $w_0 + w_1 = 1$.

In practice, the acceptance number c is fixed in advance by the decision maker (producer or consumer). Given fixed $c \geq 0$, our aim is to determine the optimal sample size the best *SASIP* that minimizes the CWR (5). Therefore, we have to solve the optimization problem

$$\min\{CWR(n, c, p_0, p_1) : \ (n, c) \in \Omega_1\}, \tag{6}$$

where $\Omega_1 = \{(n, c) : c \in \mathcal{N}_0 = \mathcal{N} \bigcup \{0\}, n \in \mathcal{N}, c \leq n\}$ is the feasible region.

Table 1 presents the sample size with minimum CWR, n_c, and their associated risks (CWR, CPR and CCR) for $r_0 = 2(1)5$, $r_1 = 1$, $f = 0.5$, $c = 0(1)3$ and $w_0 = 0.2, 0.5, 0.8$ when $q = 1.2$. In view of Table 1, the optimal sample size n_c grows when acceptance number c increases while the CWR and CPR decrease. For instance, if $r_0 = 2$ and $w_0 = 0.2$, we obtain $n_c = 5$,

Table 1: Optimal sample size, n_c with minimal CWR, and the corresponding risks (%) for selected values of c, r_0, and f when $q = 1.2$.

		$(w_0, w_1) = (0.2, 0.8)$				$(w_0, w_1) = (0.5, 0.5)$				$(w_0, w_1) = (0.8, 0.2)$			
r_0	c	n_c	CWR	CPR	CCR	n_c	CWR	CPR	CCR	n_c	CWR	CPR	CCR
2	0	5	27.156	79.828	4.582	2	38.212	47.289	29.136	1	35.372	27.398	53.978
	1	8	24.637	68.974	5.636	5	32.941	41.766	24.116	2	28.900	7.506	78.819
	2	11	22.488	61.643	5.707	8	29.248	38.193	20.304	5	26.356	13.040	57.426
	3	13	20.559	49.572	8.125	10	26.371	28.258	24.485	8	24.068	14.961	45.317
3	0	5	23.033	66.086	4.582	3	31.730	47.733	15.727	1	29.807	19.448	53.978
	1	8	18.356	48.037	5.636	6	24.117	33.108	15.126	4	23.299	17.238	37.440
	2	11	14.923	36.427	5.707	9	19.143	24.732	13.553	7	18.612	13.867	29.684
	3	14	12.291	28.297	5.431	12	15.530	19.097	11.964	10	15.140	11.135	24.485
4	0	5	19.947	55.800	4.582	3	27.228	38.729	15.727	1	26.739	15.065	53.978
	1	8	14.292	34.489	5.636	6	18.816	22.505	15.126	5	18.854	16.600	24.116
	2	12	10.528	26.643	3.622	10	13.509	18.150	8.868	8	13.533	10.631	20.304
	3	15	7.887	17.936	3.580	13	10.036	11.947	8.125	11	10.116	7.038	17.298
5	0	5	17.637	48.097	4.582	3	24.128	32.529	15.727	1	24.798	12.292	53.978
	1	9	11.525	30.544	3.374	7	15.103	20.903	9.303	5	15.443	11.726	24.116
	2	12	7.803	17.560	3.622	11	10.065	14.422	5.707	9	10.244	8.825	13.553
	3	16	5.360	12.427	2.331	14	6.871	8.311	5.431	13	7.024	6.552	8.125

$CWR = 27.156\%$, $CPR = 79.828\%$ and $CCR = 4.582\%$ when $c = 0$, whereas $n_c = 8$, $CWR = 24.637\%$, $CPR = 68.974\%$ and $CCR = 5.636\%$ when $c = 1$. Moreover, when w_0 increases, n_c and the CPR decrease, whereas the CCR grows. For instance, if $r_0 = 2,\ c = 2$, we obtain $n_c = 11$, $CWR = 22.488\%$, $CPR = 61.643\%$ and $CCR = 5.707\%$ when $w_0 = 0.2$, whereas $n_c = 8$, $CWR = 29.248\%$, $PR = 38.193\%$ and $CCR = 26.356\%$ when $w_0 = 0.5$.

5 *SASIPs* with minimal expected weighted-average risks

In designing *SASIPs*, frequentist perspectives consider only empirical data and assume a constant probability p of observing a defective or nonconforming unit in the lot under inspection. however, this assumption is not reasonable and in many situations, exists prior knowledge about the stochastic behavior of the defect rate, p , that can be incorporated to the design of sampling plans. In this section, we present a method to consider a beta distribution of p in developing an optimal failure-censored *SASIPs* for TQE lifetime distribution.

Assume that the defective rate p follows a beta $Beta(a, b)$ prior distribution, where $a, b > 0$ and its pdf is given by

$$h(p) = \frac{p^{a-1}(1-p)^{b-1}}{\mathcal{B}(1, a, b)},$$

where $\mathcal{B}(x, a, b) = \int_0^x p^{a-1}(1-p)^{b-1}dp$. Following Fernández et al. (2020), the expected producer risk (EPR) is the conditional expectation of rejecting an acceptable lot, whereas the expected consumer risk (ECR) is the conditional expectation of accepting a rejectable lot. For a given sample size m and acceptance number d, the expected producer and consumer risks are defined, respectively by

$$EPR(m, d, p_0) = E[1 - \mathcal{L}(p)|p \le p_0], \quad \text{and} \quad ECR(m, d, p_1) = E[\mathcal{L}(p)|p \ge p_1],$$

which can be expressed as

$$EPR(m, d, p_0) = 1 - \int_0^{p_0} \frac{\mathcal{L}(p)\pi(p)}{P(p \le p_0)}dp, \tag{7}$$

and

$$ECR(m,d,p_1)=\int_{p_1}^{1}\frac{\mathcal{L}(p)\pi(p)}{P(p\geq p_1)}dp, \tag{8}$$

where

$$P(p\leq p_0)=B(p_0,a,b)/B(1,a,b) \quad \text{and} \quad P(p\geq p_1)=\{B(1,a,b)-B(p_1,a,b)\}/B(1,a,b).$$

Using (4) and equations (7)-(8), the EPR and ECR are given respectively by

$$EPR(m,d,p_0)=1-\sum_{i=0}^{d}\frac{\binom{n}{i}B(p_0,a+i,b+m-i)}{B(p_0,a,b)},$$

and

$$ECR(m,d,p_1)=\sum_{i=0}^{d}\frac{\binom{n}{i}[B(1,a+i,b+m-i)-B(p_1,a+i,b+m-i)]}{[B(1,a,b)-B(p_1,a,b)]}.$$

Now, consider the expected weighted-average risk (EWR) as follows

$$EWR(m,d,p_0,p_1)=w_0EPR(m,d,p_0)+w_1ECR(m,d,p_1). \tag{9}$$

Give a fixed constant $d\geq 0$, our interest is to determine the best *SASIP* with acceptance number d, which can be obtained by minimizing (9), or equirvalently by solving the optimization problem

$$\min\{EWR(m,d,p_0,p_1):\ (m,d)\in\Omega_2\}, \tag{10}$$

where $\Omega_2=\{(m,d): d\in\mathcal{N}_0, m\in\mathcal{N}, d\leq m\}$ is the feasible region.

Table 2 shows the optimal values of m_d and the associated expected producer and consumer risks for selected values of $f=0.5$, $r_0=2(1)5$, $r_1=1$, $d=0(1)3$ when $q=1.2$. Suppose that the prior distribution is $Beta(a,b)$ with mode $5(p_0+p_1)/6$ and $a+b=5$. Since the mode of $Beta(a,b)$ is $(a-1)/\{(a+b)-2\}$, then we get $a=1+5(p_0+p_1)/2$ and $b=4-5(p_0+p_1)/2$. In light of Table 2 with $w_0=0.2$ and $d=0$, we find $m_d=3$, $EWR=17.810\%$, $EPR=47.669\%$ and $ECR=5.013\%$. It is observed that m_d tends to increase as d increases, while the EWR, EPR and ECR decrease. Moreover, m_d and EPR decrease when w_0 grows while ECR increase. It is clear that m_d tends to decrease as f increases or r_0 decrease, while EWR, EPR and ECR increase.

Figure 1 displays EWR, EPR and ECR percentages versus w_0 for $r_0=2$, $r_1=1$, $f=0.5$, $d=0$ and $q=1.2$. When w_0 increases, a decrease in EPR and increase in ECR can be observed. in contrast, the EWR has a slight growth followed by a subsequent decline.

Figure 2, plots minimal CWR and EWR sample sizes versus w_0 when $q=1.2$, $r_0=2$, $r_1=1$, $c=0$, $d=0$, $f=0.5$. This figure shows that the minimal EWR sample size, m_0, is smaller than the minimal CWR sample size, n_0. The comparison of Tables 1 and 2 shows that, in most cases, optimal sample sizes with minimal EWR are less than the corresponding minimum-CWR sample sizes.

6 Application example

We consider a real data which consist of 30 breaking strengths (in megapascals, MPa) of jute fiber measured using a gauge of length 10 mm. The data are as follows (see Xia et al. (2009) and Al-Nasser and Obeidat (2020))

43.93, 50.16, 101.15, 108.94, 123.06, 141.38, 151.48, 163.40, 177.25, 183.16
212.13, 257.44, 262.90, 291.27, 303.90, 323.83, 353.24, 376.42, 383.43, 422.11
506.60, 530.55, 590.48, 637.66, 671.49, 693.73, 700.74, 704.66, 727.23, 778.17.

Table 2: Optimal sample size, m_d with minimal EWR, and the corresponding risks (%) for selected values of d, r_0, and f when $q = 1.2$.

		$(w_0, w_1) = (0.2, 0.8)$				$(w_0, w_1) = (0.5, 0.5)$				$(w_0, w_1) = (0.8, 0.2)$			
r_0	d	m_d	EWR	EPR	ECR	m_d	EWR	EPR	ECR	m_d	EWR	EPR	ECR
2	0	3	17.810	47.669	5.013	2	23.806	35.388	12.224	1	23.602	19.816	32.435
	1	6	12.802	34.278	3.598	4	16.022	18.499	13.545	3	15.571	10.825	26.645
	2	8	9.604	21.388	4.553	7	11.911	15.918	7.904	5	11.602	6.531	23.436
	3	11	7.628	17.942	3.207	9	9.243	10.052	8.434	8	8.808	6.801	13.489
3	0	3	14.563	35.596	5.549	2	19.434	25.596	13.271	1	19.942	13.841	34.178
	1	6	8.926	20.286	4.057	5	11.387	15.019	7.754	4	11.490	10.031	14.894
	2	9	5.983	12.993	2.979	8	7.470	9.786	5.155	7	7.507	6.923	8.869
	3	12	4.221	8.827	2.246	11	5.196	6.727	3.665	10	5.210	4.901	5.931
4	0	4	12.507	35.679	2.577	2	16.917	20.016	13.819	1	17.956	10.625	35.062
	1	7	6.738	17.154	2.274	6	8.790	13.273	4.307	4	9.061	6.255	15.608
	2	10	4.049	9.214	1.835	9	5.132	7.083	3.181	8	5.279	5.191	5.483
	3	13	2.606	5.263	1.468	12	3.223	4.039	2.407	11	3.266	2.987	3.918
5	0	4	10.837	29.905	2.666	3	14.753	23.494	6.011	2	15.743	16.423	14.155
	1	7	5.322	12.229	2.362	6	6.894	9.326	4.463	5	7.187	6.643	8.454
	2	11	2.920	7.166	1.100	10	3.767	5.623	1.912	8	3.849	3.061	5.689
	3	14	1.716	3.554	0.929	13	2.146	2.760	1.532	12	2.208	2.079	2.509

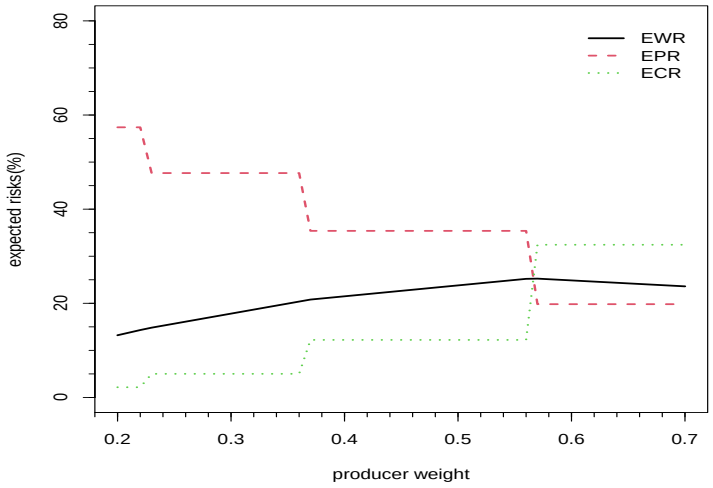


Figure 1: EWR, EPR and ECR percentages versus w_0 for $r_0 = 2,\ r_1 = 1,\ f = 0.5,\ d = 0$ when $q = 1.2$.

The maximum likelihood (ML) estimates of the scale and shape parameters are calculated as $\hat{\lambda} = 0.0049$ and $\hat{q} = 1.3485$, respectively. A Kolmogorov-Smirnov (K-S) test is employed to check that the TQE distribution fits the aforementioned data well. The K-S statistic is computed as $D = 0.2058$ with the corresponding p-value that equals 0.1362. Thus, we conclude that the TQE distribution has good fit on considered data set.

Using (3), the mean life can be estimated as $\hat{\mu} = [\hat{\lambda}(3-2\hat{q})]^{-1} = 673.54$. Consider now that the termination ratio is $f = 0.5$. Then, the termination time is $t = f\mu_0 = 336.77$. Assume that $r_0 = 2, d = 5$ and $w_0 = 0.5$. From Table 3, we get $p_0 \equiv p_0(0.5, 1.3485, 2) = 0.3670$ and $p_1 \equiv p_1(0.5, 1.3485, 1) = 0.5627$ and then $(a, b) \equiv (a_{p_0,p_1}, b_{p_0,p_1}) = (3.3242, 1.6758)$. Moreover, the optimal EWR sample size is $m_5 = 11$ with corresponding risks $EWR = 7.3850$, $EPR = 7.6350$ and $ECR = 7.1350$. According to these specifications, $m_5 = 11$ fibres selected randomly from the lot must be put on test for $t = 336.77$ MPa. The lot will be accepted if

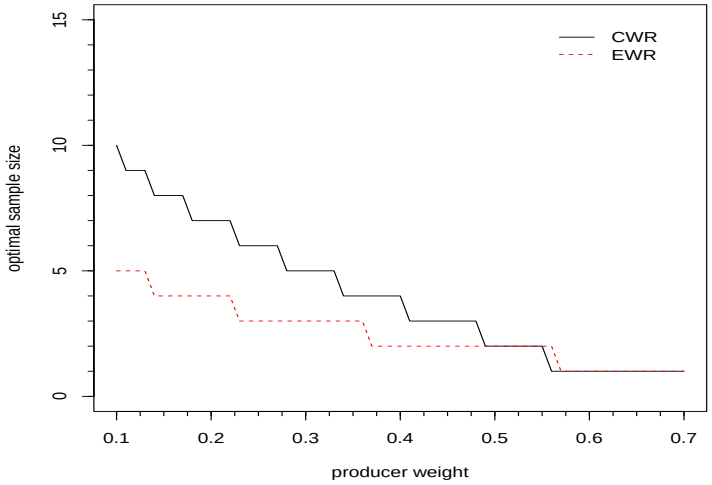


Figure 2: Optimal sample sizes with minimal CWR and EWR versus w_0 when $q = 1.2,\ r_1 = 1,\ c = 0,\ d = 0,\ f = 0.5$ and $r_0 = 2$.

Table 3: Minimum EWR sample sizes for selected values of $w_0 = 0.5, d = 5$ and $f = 0.5$ when $q = 1.34$ and $p \sim Beta(a, b)$ with mode $5(p_0 + p_1)/6$ and $a + b = 5$.

r_0	p_0	p_1	a	b	m_d	EWR	EPR	ECR
2	0.3670	0.5627	3.3242	1.6758	11	7.3850	7.6350	7.1350
3	0.2713	0.5627	3.0850	1.9150	13	3.6980	4.5140	2.8820
4	0.2150	0.5627	2.9442	2.0558	14	2.0480	2.2870	1.8090
5	0.1780	0.5627	2.8517	2.1483	15	1.2180	1.3540	1.0810

the number of failures is at most $d = 5$.

7 Concluding remarks

The probability p of observing defects in a lot inspection is usually assumed to be constant. There are many practical situations in the industry in which exists prior knowledge about the stochastic behavior of the defect rate that can be incorporated to the design of sampling plans. An approch to determine optimal WR $c-$defective plan (n_c, c) and optimal EWR $d-$defective plan, (m_d, d) under time-censoring presented in this paper. Constrained optimization problems have been formulated to deremine the optimal plans. The results show that, in most cases, the minimum EWR sample sizes and expected risks are less than the corresponding minimum WR sample sizes and classical risks.

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