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Design of LQR Controller with Infinite Horizon for Delayed Discrete Systems based on Predictive Control and Reduction Method

Sara Mahmoudi Rashid, Ph.D. student in Electrical Engineering Department
University of Tabriz, Tabriz, Iran

Abstract— **In this paper, the linear quadratic regulator controller is designed for a specific class of discrete time-delay systems. The controller, which can satisfy the constraints and compensate for the destructive effects while satisfying the constraints, has attracted much attention. The presence of time-varying delays at local control inputs, which can destabilize the system and disrupt the process, has also been investigated. A satellite orbit system is considered that corrects the position by a predictive control method based on the constrained model. The aim is to design the linear quadratic regulator controller based on the predictive controller by the reduction method so that the closed-loop system is asymptotically stable for all acceptable uncertainties and time delays and minimizes the effect of external disturbances. This requires a high speed and accurate controller to be able to meet the desired control objectives as much as possible. Also, by the theory of stability proof theory with Lyapunov krasovskii function and in the form of linear matrix inequalities, conditions independent of the delay for establishing exponential convergence are obtained. Finally, the effectiveness of the results obtained from the proposed control scheme is demonstrated by a numerical example of the position of the satellite in order to keep it in orbit and reduce fuel consumption.**

Keywords: LQR controller, Time-varying time delay, Lyapunov Krasovskii Function, Reduction method, Predictive controller.

1. Main text

Satellites are influenced by the forces that cause the satellite to move away from the orbit for which it was designed. To counter these forces, satellites in orbit are equipped with control propellants to compensate for these constraints and keep the satellite in orbit. On the other hand, due to the lifespan of satellites, which are several years old, a large amount of fuel mass is required, which shows the importance of optimal fuel consumption. Of course, it should be noted that another reason for optimal fuel consumption is the high cost of launching satellites per mass. Also, due to design limitations, the mass of fuel along with the satellite should not exceed a certain amount. Automatic control of the satellite orbit is that the satellite itself can maintain its position. Automatic control will reduce costs and increase mission accuracy, eliminating the need for an equipped and advanced ground station, for example. Also, by using automatic orbital control, the need for fuel will be reduced ,and the weight of the satellite will be reduced, which will reduce the cost of launching and building the satellite [۱].

Predictive control is an optimal control strategy that minimizes the cost function for the system by predicting the dynamic model of the system. This control can simultaneously achieve several control objectives by considering the constraints on the input and output of the system and ensuring optimization. This control method can be implemented online. Due to the limitations of satellite fuel, which leads to optimal fuel consumption, as well as the constraints on the amount of propulsion power of the system output. Model-based predictive control is a good choice for automatic satellite orbit control. Predictive control based on the linearization of the dynamic equations governing the satellite in space revolves around the main orbit. The cost function is considered to be square, which will reduce the volume of calculations to find the optimal point. The simulations show that the combined predictor control with linear quadratic regulator (LQR), by considering the constraints on the input and output of the system, has significantly reduced fuel consumption compared to the linear quadratic regulator only [۲].

Nowadays, the issue of examining the limitations and controlling the system in the presence of these constraints is one of the important and widely used issues in theoretical research and practical activities. Another issue that challenges the controller design is the issue of delay, which is unavoidable in many physical systems and is a major cause of instability in systems [۳]. Delayed systems can be classified into input delay, state delay, and output delay, which may be single or multiple delays. Recently, systems with delayed control input have received more attention .[٤] According to the Lyapunov Krasovskii function (LKF) method, stability conditions are usually obtained by linear matrix inequalities (LMI) ,which can be solved using various computational tools. Although a number of studies have analyzed stability specifically for systems with time-varying delay at input, ideas for improving stability and scrutiny have not been used. The purpose of the design is to create a control system that changes in the current state of the system have the least effect on the output.

In recent years, the issue of the stability of delayed linear systems has been investigated. In these papers, some sufficient conditions for proving the stability in delay are presented by Lyapunov inequality methods or Ricketts inequalities [۹-۵]. In [10], the prediction control model for a class of fuzzy discrete systems that are subject to variable time-delayed delays and perturbations is studied. The proposed method is razumikhin, which includes a Lyapunov function related to state space not added to the main dynamics of the system compared to the Krasovskii method. The system considered in this paper is decomposed into several subsystems, each of which is represented by a fuzzy Takagi-Sugeno (T-S) model, and the relationship between the two subsystems is also considered. Since the main part of the model prediction control is optimization, the hierarchical design for the optimization problem is performed by the linear matrix inequality (LMIs) method. Thus the closed loop system is asymptotically stable. [11] provides a general framework for the LQR-controlled system, taking into account uncertainties, perturbations, and time delays. Sufficient conditions are provided by Lyapunov-Krasovskii function and Lipschitz one-sided condition and quadratic internal boundary inequality to ensure asymptotic stability. A neural controller with predictive controller is also provided for a class of delayed systems in the presence of unknown dead zone, external perturbations, and stimulus faults in [12]. In this paper, Lyapunov-Krasovskii quadratic functions for dealing with system delays are introduced. Unknown system functions are estimated using radial basis neural networks. The delimitation of all closed-loop signals is ensured by Lyapunov analysis and it is proved that the tracking errors converge to a small area of origin. Therefore, the issue of proving stability for delayed systems must be considered, as to date, very few stability results are available in the researched researches with the aim of reducing costs. As a result, it is essential to provide innovative design methods for multipurpose controllers in order to combine multiple controllers in order to improve performance.

Compared to existing studies, this is the main contribution of this article. In the second part, mathematical modeling of satellite motion in orbit is described. In the third stage, the LQR controller problem is solved based on the predictive control and the method of reducing the conditions independent of the improved delay. As a result, the stability of the closed-loop system and in a comprehensive structure, the stability of the system is exerted by matrix inequalities. In the fourth section, and in fact the simulation of the article, the simulation results are presented. Finally, in the fifth and final part of the article, the general results of the article are stated.

2. Mathematical modeling of satellite motion in orbit

In this section, the orbital dynamics equations on the Earth satellite shown in Figure 1 are expressed as (1).

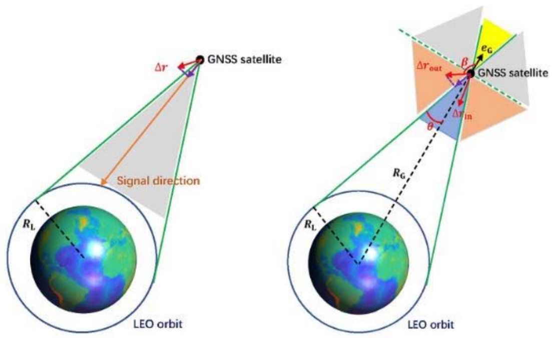

Figure 1. Physical interpretations of orbital satellite elements [۱۰]

$$S:\begin{cases} \delta\ddot{x} - 3n^2\delta x - 2n\delta\dot{y} = \dfrac{1}{m}F_x + \alpha_{p_x} \\ \delta\ddot{y} + 2n\delta\dot{x} = \dfrac{1}{m}F_y + \alpha_{p_y} \\ \delta\ddot{z} + n^2\delta z = \dfrac{1}{m}F_z + \alpha_{p_z} \end{cases} \qquad (۱)$$

Where F_x, F_y and F_z are the components of the control forces acting on the satellite by the propellants, α_{p_x} , α_{p_y} and α_{p_z}accelerating the satellite and m the mass of the satellite. Also, according to the relation $n = \sqrt{(\mu/r^{\wedge}3\,)}$, r is the distance of the satellite from the center of the earth and μ is the constant of gravity. These linear equations are the difference between the motion of the satellite and the main orbit, the purpose of the control design is to reduce this difference to zero and keep the satellite in the reference orbit, ie zero error. Considering the state variables as $x = [\delta x\ \delta y\ \delta z\ \delta\dot{x}\ \delta\dot{y}\ \delta\dot{z}]$ and the control input vector $u = [F_x\ F_y\ F_z]$, the state space equations of the linear system are (2).

$$\dot{x} = Ax + Bu,\ \ y = Cx,\ A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & \frac{1}{m} & 0 \\ 0 & 0 & \frac{1}{m} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad (۲)$$

In order to implement predictive control, the system equations must be expressed discretely, which is done by the Euler step-forward method, which is a first-order method.

3. Infinite horizon LQR for delayed discrete systems based on predictive controller

The purpose of designing a satellite orbit control is to return the satellite to its original orbit, minimizing fuel consumption and reducing power. To achieve the stated goals, a model-based predictive control is designed. The control method ensures optimality by considering the performance and constraints on the system. Predictive control by solving the problem of optimal control and predicting the future, it generates a control signal, which is done by considering the cost function and constraints on the input and state of the system at any time step. Is accepted in such a way that at each time step, it calculates a sequence of control inputs for future moments of the system and applies the first sentence as input to the system using the reducing horizon method, and after measuring The output and the error between the system and the desired state will be repeated in the next time step of this process until the error rate reaches zero or an acceptable value. By linearly considering the equations, the constraints being equal and unequal, and the cost function being quadratic, the prediction control will be formulated as a quadratic problem that has a small amount of computation to implement online [۱۱].

Consider a discrete linear system for the N stabilization problem of the input delay subsystem by differential equations (3) with fixed matrices and constant delay h_{ij}.

$$x(k+1) = Ax(k) + Bu(k-h) \tag{۳}$$

Where $k \in Z_{+}$, $\mathrm{h} \in Z_{+}$and $x(k) \in R^{n_i}$ are the state vectors and $u_i(k) \in R^{n_{ui}}$ are the control inputs of the system. $A_i \in \mathbb{R}^{n_i \times n_i}$ and $B_i \in \mathbb{R}^{n_i \times n_{ui}}$ are nominal fixed matrices with appropriate dimensions.

The initial conditions and in fact the primary function are considered as relation (4).

$$\begin{aligned} &col\{x_i(0), u_i(-1), \ldots, u_i(-h), x_j(0), u_j(-1), \ldots, u_j(-h)\} = \\ &col\{\varphi_i(0), 0, \ldots, 0, \varphi_j(0), 0, \ldots, 0\} \end{aligned} \tag{٤}$$

By the additive method [16], the additive state vector can be considered as (5).

$$x_{aug}(k) = col\{x_i(k), u_i(k-1), \ldots, u_i(k-h), x_j(k), u_j(k-1), \ldots, u_j(k-h)\} \tag{٥}$$

In this case, the delayed system becomes a system without delay (6).

$$x_{aug}(k+1) = A_{aug} x_{aug}(k) + B_{aug} u(k), \quad \mathrm{k} \in Z_{+}, \quad \mathrm{x}_{aug}(k) \in \mathbb{R}^{(h_{ij}+1)(n_i+n_j)}, \tag{٦}$$

$$A_{aug} = \begin{bmatrix} A_i & 0 & . & . & . & A_{ij} \\ I_n & 0 & . & . & . & 0 \\ . & . & & & & . \\ . & & . & & & . \\ . & & & . & & . \\ 0 & . & . & . & I_n & 0 \end{bmatrix}, \qquad \mathrm{B}_{aug} = \begin{bmatrix} B_i & B_{ij} \\ 0 & 0 \\ . & . \\ . & . \\ . & . \\ 0 & 0 \end{bmatrix}$$

It should be noted that the A_{aug} matrix is unique. Consider the LQR problem with infinite horizon (7) considering the initial conditions (4). Predictive control is a constrained optimal control that ensures optimization of the system despite the constraints on the inputs and dynamics. Constraints are applied systematically in predictive control, and as the complexity of the constraints increases, the controller's resistance to uncertainties and perturbations will increase. The cost function for this controller is of the square type, which includes the control input and output terms of the system, so that by minimizing fuel consumption, the accuracy of following the reference path is increased and the error caused by Disturb the forces to be compensated. The goal is to find the control law $\boldsymbol{u}$ in such a way that the stable system becomes asymptotic and becomes a quadratic cost function. Q and R are square weight matrices for setting cost function parameters, which increase the control accuracy and reduce energy consumption by adjusting these matrices. The cost function consists of two sentences, the first of which is to consider the system error between the current state and the output of the system, and the second of which is to minimize energy consumption. To evaluate the performance of the predictive controller, a second-order regulator controller is considered. The cost function considered for the second-order regulator is (7).

$$J_{ih} = \sum_{k=0}^{\infty}[x_i^T(k)Q_i x_i(k) + u_i^T(k-h)R_i u_i(k-h)] + \sum_{\substack{j=1,\\ j\neq i}}^{N}\sum_{k=0}^{\infty}[x_{ij}^T(k)Q_{ij}x_{ij}(k) + u_{ij}^T(k-h)R_{ij}u_{ij}(k-h)], Q_i \geq 0, Q_{ij} \geq 0, R_i > 0, R_{ij} > 0 \quad (٧)$$

As a result, with the additional method, the delayed LQR problem becomes a delayed LQR problem. The LQR problem for a system without delay is that if the matrix pair (A_{aug}, B_{aug}) is stable and the matrix pair (A_{aug}, Q_{aug}) is observable, then an optimal control rule for the system without delay is (8) there is. Of course, it should be noted that if the matrix pair (A, B) (for $h_{ij} = 0$))) is stable, then the matrix pair (A_{aug}, B_{aug}) will be stable.

$$u_i(k) = K_i x_{aug}(k) + \sum_{\substack{j=1,\\ j\neq i}}^{N} K_{ij}x_{aug}(k), \quad K_i + \sum_{\substack{j=1,\\ j\neq i}}^{N} K_{ij} = -(B_{aug}^T P B_{aug} + R_{aug})^{-1} B_{aug}^T P A_{aug}, \quad P \in \mathbb{R}^{(h_{ij}+1)(n_i+n_j)\times(h_{ij}+1)(n_i+n_j)} \quad (٨)$$

That the matrix P is a unique positive response given by the time-discrete algebraic Richter equation (DARE) (9).

$$P = A_{aug}^T[P - PB_{aug}(B_{aug}^T PB_{aug} + R_{aug})^{-1}B_{aug}^T P]A_{aug} + Q_{aug}, \quad (٩)$$

$$R_{aug} = \begin{bmatrix} R_i & . & . & . & R_N \\ & . & & & \\ & & R_j & & \\ & & & . & \\ & & & & R_N \end{bmatrix}, Q_{aug} = \begin{bmatrix} Q_i & . & . & . & Q_N \\ & . & & & \\ & & Q_j & & \\ & & & . & \\ & & & & Q_N \end{bmatrix}$$

In which case the matrix $A_{aug} + B_{aug}[K_i, K_{ij}]$ is stable. This leads to optimal feedback (10) with distributed delay for the main system.

$$u(k) = Kcol\{x_i(k), u_i(k-1), ..., u_i(k-h), u_j(k), u_j(k-1), ..., u_j(k-h)\} \quad (١٠)$$

This result is obtained from the Hautus Criterion [17] with the following proof.

Proof: The following matrix has full rank for all $|s| \geq 1$ because for $|s| \geq 1$ the matrix $[sI_n - A \quad B]$ has full rank for the extension system. In this case, the main system and in fact the system are stable without delay.

$$[SI - A_{aug}B_{aug}] = \begin{bmatrix} SI_{n_i} - A_i & SI_{n_{ij}} - A_{ij} & 0 & 0 & 0 & 0 & 0 & -B_i & -B_{ij} & 0 \\ 0 & SI_{n_{ui}} & SI_{n_{uj}} & 0 & 0 & 0 & 0 & 0 & I_{n_{ui}} & I_{n_{uj}} \\ . & -I_{n_{ui}} & -I_{n_{uj}} & . & . & . & 0 & 0 & 0 & 0 \\ . & & . & & . & & & & & \\ . & & & . & & . & & & & \end{bmatrix} \quad (١١)$$

Therefore, if the matrices $(A, B, \sqrt{Q})$ are stable and observable (for $h = 0$), then there exists a unique optimal controller in form (10) in which the definite unique positive response P is obtained from the discrete algebraic Richter equation. Optimal control (8) has a distributed delay at the input that, like the controller, is based on a forecast for a continuous system. Assuming the system is considered and the initial function and cost function and application of reduction method, which is mainly considered for input delay systems, to derive a reduced response answer to the LQR problem with input delay, from the design discussion on the basis of prediction is used. By defining $v(k) = u(k-h)$ the cost function J_h can be written as (12).

$$J_{ih} = \sum_{M=0}^{h-1} [\varphi_i^T(0)(A_i^T)^M Q_i A_i^M \varphi_i] + \sum_{\substack{j=1,\\ j\neq i}}^{N} \sum_{M=0}^{h-1} [\varphi_{ij}^T(0)(A_{ij}^T)^M Q_{ij} A_{ij}^M \varphi_{ij}(0)] + \bar{J}_{ih} \quad (۱۲)$$

$$\bar{J}_{ih} = \sum_{k=h}^{\infty} [x_i^T(k) Q_i x_i(k) + v_i^T(k) R_i v_i(k)] + \sum_{\substack{j=1,\\ j\neq i}}^{N} \sum_{k=h}^{\infty} [x_{ij}^T(k) Q_{ij} x_{ij}(k) + v_{ij}^T(k) R_{ij} v_{ij}(k)]$$

According to Equations (12), J_{ih} sentences contain fixed functions that are known and cannot be changed. Functions such as control inputs that can be changed for design appear in $\bar{J}_{ih}$ sentences. Thus the J_{ih} minimization for the above system leads to the $\bar{J}_{ih}$ minimization in the system direction without delay by defining the v function based on u.

$$x_i(k+1) = A_i x_i(k) + B_i v_i(k) + \sum_{\substack{j=1,\\ i\neq j}}^{N} [A_{ij} x_j(k) + B_{ij} v_j(k)], \quad (۱۳)$$

$$k \geq h_{ij}, \quad x_i(h_{ij}) = A_i^{h_{ij}} \varphi_i(0) + \sum_{\substack{j=1,\\ i\neq j}}^{N} A_{ij}^{h_{ij}} \varphi_{ij}(0)$$

Now if the matrix pair (A, B) is stable and the matrix pair $(A, \sqrt{Q})$ is observable, then the unique optimal control of the latter problem will be (14).

$$v_i(k) = u_i(k-h), \quad K_i + \sum_{\substack{j=1,\\ j\neq i}}^{N} K_{ij} = (-B_{aug}^T P B_{aug} + R_{aug})^{-1} B_{aug}^T P A_{aug}, P \in \mathbb{R}^{(h_{ij}+1)(n_i+n_j)\times(h_{ij}+} \quad (۱٤)$$

Where P is the unique non-negative response of the discrete algebraic rickety equation. In addition, optimal feedback stabilizes the closed-loop system. Now an important point to note is that with the discussions done, the function v is obtained, which with a few modifications can be obtained to control the function u according to Equation (15).

$$u_i(k) = v_i(k+h) = K_i + \sum_{\substack{j=1,\\ j\neq i}}^{N} K_{ij} x(k+h_{ij}), x(k+h) = A^h x(k) + \sum_{j=k}^{k+h-1} A^{k+h-j-1} B u(j-h) \quad (۱٥)$$

By changing the upper and lower limits of Sigma in Equation (15), Equation (16) is obtained.

$$u(k) = K[A^h x(k) + \sum_{j=-h}^{-1} A^{-j-1} B u(k+j)] \quad (۱٦)$$

Also, the optimal value of J_h^* as (17) is obtained by the optimal control.

$$J_h^* = \sum_{j=0}^{h-1} \varphi^T(0)(A^j)^T Q A^j \varphi(0) + \varphi^T(0)(A^h)^T P A^h \varphi(0) \quad (۱۷)$$

The method of reduction to discrete indefinite linear systems with delayed input has also been developed. Consider the main system with indefinite and non-small delays $\tau_k = h + \eta_k$.

$$x(k+1) = Ax(k) + Bu(k-\tau_k), \quad x(k) \in \mathbb{R}^n, \quad u(k) \in \mathbb{R}^{n_u}, \quad k=0,1,... \tag{۱۹}$$

In this system, h is a fixed nominal value and $|\eta_k| \le \mu < |h$ indicates an indefinite delay. By changing the variable, a system relation (20) is obtained.

$$z(k) = A^h x(k) + \sum_{j=-h}^{-1} A^{-j-1} Bu(k+j), \tag{۲۰}$$
$$z(k+1) = Az(k) + Bu(k) + A^h B[u(k-\tau_k) - u(k-h)]$$

In this case, due to the variability of the delay in the system, the direct reduction method can not be used, so the variable change method is considered to solve the continuation of the problem. Now suppose there exists an interest K such that the matrix $A + BK$ is stable. In this case, the feedback $u(k) = Kz(k)$ stabilizes the above system if system (21) is stable.

$$z(k+1) = (A+BK)z(k) + A^h BK[z(k-\tau_k) - z(k-h)] \tag{۲۱}$$

System stability can be analyzed using the Lyapunov Krasovskii method for systems with non-small latency. Therefore LKF (22) can be considered for the new system.

$$V(k) = z^T(k)Pz(k) + \sum_{j=-\mu}^{\mu-1} \sum_{s=k+j-h}^{k-1} \xi^T(s) R_1 \xi(s), \xi(s) = z(s+1) - z(s), \quad P>0, \quad R_1 > 0 \tag{۲۲}$$

The issue of Guaranteed Cost Control (GCC) for systems with variable delay indefinitely $\tau_k \in [0, h]$ and fixed matrices A, B are examined. Now consider the cost function as (23).

$$J = \sum_{k=0}^{\infty} z^T(k)z(k), \quad z(k) \in \mathbb{R}^{n_z}, \quad z(k)=Lx(k)+Du(k-\tau_k) \tag{۲۳}$$

For delayed systems or indeterminate matrices, optimal control and optimal cost (such as LQR) cannot be achieved. Instead, with the given initial condition x_0, a control rule can be found that for all uncertainties reaches a guaranteed minimum cost of δ for J, i.e. $J \le \delta$. A feedback control mode $u(k) = Kx(k)$ is provided for an initial condition x_0 that for all indefinite delays τ_k the minimum guaranteed cost value $J(x_0) \le \delta$ can be achieved. The closed loop system is in the form of (24).

$$x(k+1) = Ax(k) + BKx(k-\tau_k), \quad z(k)=Lx(k)+Du(k-\tau_k) \tag{۲۴}$$

Proof: Consider the standard Lyapunov candidate function (25) for the exponential stability of the system with $\tau_{ij}(t) \in [0, h_{ij}]$ (δ is scalar). Suppose $V: [t_{ij0}, \infty) \to \mathbb{R}_+$ is an absolutely continuous local function. If $\delta >$ 0exists in such a way that for all $t_{ij} \ge t_{ij0}$ there is a system relation (25).

$$V(x_k) = V_P(k) + V_S(k) + V_R(k), \qquad P>0, R \ge 0, S \ge 0 \tag{۲۵}$$
$$V_P(k) = x^T(k)Px(k), V_S(k) = \sum_{j=k-h}^{k-1} x^T(j)Sx(j),$$
$$V_R(k) = h \sum_{m=-h}^{-1} \sum_{j=k+m}^{k-1} \bar{y}^T(j) R \bar{y}(j), \quad \bar{y}(j) = x(j+1) - x(j)$$

Now if it finds $\alpha > 0$ that negates the expression (26), then the closed-loop system becomes stable.

$$V(x_{k+1}) - V(x_k) + z^T(k)z(k) \le \bar{\xi}^T(k) \Gamma \bar{\xi}(k) < -\alpha |x(k)|^2, \alpha > 0 \tag{۲۶}$$

In this section, by considering the addition vector as (27), the desired LMI is obtained using Schur complement [12].

$$\bar{\xi}(k) = col\{x(k), y(k), x(k-h), x(k-\tau_k)\} \tag{۲۷}$$

$$\Gamma = \begin{bmatrix} & | & L^T \\ \phi & | & 0 \\ & | & K^T D^T \\ - & - & - \\ * & | & -I \end{bmatrix} < 0, \phi = \begin{bmatrix} \phi_{11} & \phi_{12} & S_{12} & R - S_{12} + P_2^T A_1 \\ * & \phi_{22} & 0 & P_3^T A_1 \\ * & * & -(S+R) & R - S_{12}^T \\ * & * & * & -2R + S_{12} + S_{12}^T \end{bmatrix}, A_1 = BK \tag{۲۸}$$

$$\phi_{11} = (A^T - I)P_2 + P_2^T(A - I) + S - R,$$
$$\phi_{12} = P - P_2^T + (A^T - I)P_3,$$
$$\phi_{22} = -P_3 - P_3^T + P + h^2 R$$

As a result, by taking Sigma from the unequal sides from *0* to *N*, the guaranteed cost function (29) is obtained.

$$J = \sum_{k=0}^{N} z^T(k) z(k) \le V(x_0) - V(x_{N+1}) \le V(x_0), \tag{۲۹}$$
$$J \le V(x_0) = x^T(0) P x(0)$$

4. Simulation

In this section, the performance of the model-based predictive control in maintaining the satellite on the reference orbit and combining it with the quadratic regulator control is shown linearly. The specifications of the reference circuit for implementing predictive control and physical parameters of the satellite are $m = 100kg$, $\mu = 2.5$ and $r_0 = 1.3m$. By applying LMI and $h = 3$ and $\varepsilon = 1$ and the initial condition $\varphi(-k) = e^{-k}, k \le 0$of the high cost band equal to $\delta = 0.0716$ is obtained. The main goal is to eliminate the initial error and keep the satellite on the reference circuit with the highest accuracy and minimize fuel consumption. Q and R are weight matrices designed to control the control with values of $R = diag(0.072\ 0.072\ 0.072)$ and $Q = diag(0.023\ 0.023\ 0.023)$, also *N = 10* is the forecast horizon.

Figure 2 shows the error between the satellite and the reference circuit in the design of the LQR controller by the predictor controller and the reduction method. The initial error is eliminated after about 30 seconds and the satellite is placed in orbit with high accuracy. The constraint, which included preventing the satellite from decreasing relative to the reference orbit, was applied in the controller design.

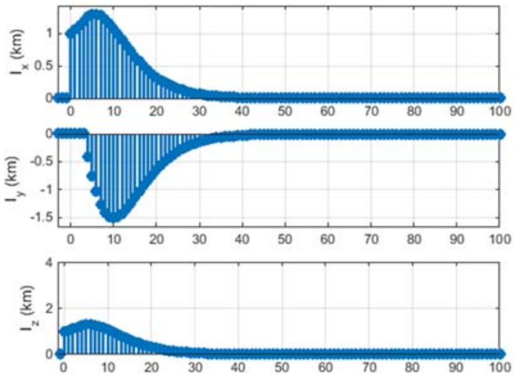

Figure 2: Error between satellite and reference orbit

Figure 3 also shows the control force applied to return the satellite to its original orbit and maintain it. The maximum force exerted by the forecast control is 0.1 N, which indicates that the constraint on the control input is taken into account and significantly reduced. Due to the satellite disturbances, it moves out of the orbital plane, but when it reaches the maximum allowable distance, the controller is activated and returns the satellite to the main orbit with very little force.

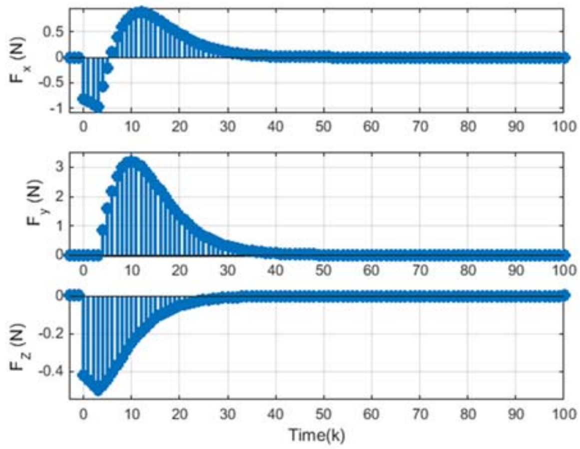

Figure 3: Control force applied to return the satellite to the main orbit

5. Conclusion

In this paper, the automatic control of the orbit of low altitude satellites using the LQR controller is performed by the predictor controller based on the reduction method. Predictive control calculates the optimal control input to apply to the system by solving the finite horizon optimization problem by considering the constraints on the delayed system. The superiority of this method over individual predictive control and optimal linear quadratic regulator control in reducing applied force, reducing fuel consumption and considering the existing physical constraints has been shown. Using the Lyapunov function, the stability of the closed-loop system is proved by the linear matrix inequality tool. The proposed design is practical and at the same time has few calculations, which could potentially provide a cost-effective solution for estimating delayed discrete systems. The simulation results show the reliability of the performance design in estimating the optimal state functions, which ensures the exponential convergence of the error. Comparing the resulting responses is the good performance of the proposed controller to meet the objectives of the main system. Future studies can also address the incidence of perturbations and stimulus saturation and uncertain consideration for system parameters and their effects.

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