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PMSM Drive Control Based on Constrained Optimal Model Predictive Method

Hassan Mohammadi Pirouz[1], **Mehdi Fadaee**[2], **Hadi Zahmatkesh Hesari**[3]

[1]Department of Electrical Engineering, University of Bojnord, Bojnord; Email: pirouz@ub.ac.ir
[2]Department of Electrical Engineering, Azad University of Oloom Tahghighat, Tehran
[3]Faculty of Electrical Engineering, Shahid Beheshti University, Tehran

Abstract
This paper investigates a new model-based predictive method based on the Pontryagin Maximum Principle, to control the speed and position of permanent magnet synchronous motors. In the proposed controller, the linearized mathematical model of the motor during each switching time is used to predict the future behavior of the control variables. In order to improve the dynamic performance of the drive system, a new optimization algorithm is proposed in which the optimal control laws are obtained by an offline multi-parameter optimization. Therefore, the volume of online calculations of the processor can be significantly reduced, providing the possibility for controlling the drive system with a low-cost microcontroller. Besides, voltage and current constrains are applied independently in two separate control loops to eliminate the complexity of the computation and the limitations of the load variation range. The performance of the proposed control strategy has been evaluated through processor-in-loop experiments, and performed by a digital signal processor which is commonly used in the motor drive controllers. The results show that suggested model predictive control can improve the speed dynamics of a traction motor when the load torque changes suddenly, compared to conventional predictive methods.

Keywords: PMSM, MPC, predictive control method, speed and torque control, voltage and current constrains

Introduction

Due to recent advances in real-time signal processing, various methods of MPC have been developed and successfully applied to electric machine drive systems. The Finite-Set MPC technique uses a limited number of voltage vectors to evaluate the cost function. In general, some benefits of this approach are the simplicity of the design process, no need for voltage modulator, online relatively optimization, and the simplicity in managing constraints and nonlinearities of the system [1]. However, the obtained voltage vector cannot result in an absolute minimum value of the cost function, even with a long calculation time or high switching frequency. Besides, system constraints do not effectively affect the process of designating a voltage vector as the control signal [2].

In another approach, the cost function is calculated through an optimal control method, by considering the system model, system initial conditions, and some constraints on control variables and inputs [3]. Hence the control signals can be obtained as a continuous and precise analytical function. But, real-time solving of optimal control problems is time-consuming and results in a high volume of computation over each sampling time interval. In this regard, reference [4-5] used an optimization window to solve the constrained optimal control problem. The necessary condition to get an answer in this way is the reversibility of the Hessian matrix in each prediction horizon, whereas the calculation and evaluation of this matrix increase the computing time. Additionally, system constraints are examined after calculating the control signals in each control horizon, and by applying receding horizon control, this increases the probability that control signals exceed the constraints of the drive system. To solve this problem, researchers calculated the value of the Hessian matrix at the steady-state condition, though no optimal performance is observed in transient conditions.

In references [6-7], the active set method and the Hilderth’s method have been used in the PMSM drive system, to solve the online optimization problem, which both require considerable time to identify the active constraints. Recently, Bemporad et al. had presented the explicit linear quadratic optimization method for solving a constrained optimal control problem, in which the offline control signals are determined based on the position of control variables in the state-space polyhedron as a set of piecewise affine functions of the state variables vectors [8]. In this method, the trajectory of control variables is designed as a map, depending on the initial condition and the final state values of the system, so that this map leads to an optimal control path. Although, with any change in the steady-state conditions, the map will no longer be optimal. Reference [9] has applied this mapping method to control the speed of a PMSM in no-load conditions, and showed that the computational speed can be greatly improved, however, under the predetermined conditions. In another application reference [10] used a trajectory map as a part of the cascaded control process to eliminate the offset error caused by the current sensors in the speed control of PMSM. Since finding control variables in the polyhedron is a time-consuming process, reference [11] provides some fast search strategies, although because of increasing the vector dimensions of control variables, determining the active area conditions in the polyhedral state-space will be more complex.

This paper presents a comprehensive algorithm for designing speed and current controllers of the PMSM drive system so that it can generate control signals offline, and as a set of linear functions of the state variables and inputs, to bring significant savings in computing time. The proposed algorithm employs a cascaded structure in which the speed controller is performed based on the optimization window technique, and the current controller is designed on the basis of the Pontryagin maximum principle. These designs are not just for steady-state conditions, and the system constraints are also particularly effective in the process of generating control signals. The feasibility of the control process in the real working conditions is vitally important; therefore, the necessary and sufficient condition for generating control signals by planned algorithm is also presented analytically.

Optimal Model Predictive Control Based on the Pontryagin Maximum Principle

This section presents the algorithm which is used for solving the optimization problem in this paper based on the Pontryagin maximum principle [12].

Suppose that the nonlinear model of the system, which is going to be controlled around an operating point at the moment t_i, is linearized by using a Taylor's series expansion as follows:

$$\begin{aligned} X^{\bullet}(t) &= f(X(t), U(t), t) = A(X(t_i), t_i)X(t) + B(t_i)U(t) + D(X(t_i), t_i) \\ X_{min} &\le X(t) \le X_{max} \\ U_{min} &\le U(t) \le U_{max} \end{aligned} \quad (1)$$

in which, $X(t)$ and $U(t)$ are the machine state vector and system input vector respectively. Moreover, $A(X(t_i), t_i)$ is the state matrix, $B(t_i)$ is the input matrix and $D(X(t_i), t_i)$ is the matrix of constant statements caused by linearization and uncontrollable inputs, all at t_i. Then, in order for the expressed control system of (1) to be able to get to the optimal conditions from the current situation, a performance index is considered as follows:

$$\mathcal{J} = Min\frac{1}{2}\left[S\big(X^*, X(t_f)\big) + \int_{t_i}^{t_f} V(X(\tau), X^*, \tau)d\tau\right] \quad (2)$$

where X^* is the desired state-vector at the end of the predictive horizon (at time t_f) and $X(t_f)$ is the state-vector at t_f. The terminal function of $S\big(X^*, X(t_f)\big)$ is to minimize the terminal state error and the integral function of $V(X(t), X^*, t)$ is to optimize the tracking path. Based on the purpose of the performance index, the functions of S and V are defined as follows:

$$\begin{aligned} S &= \big(X(t_f) - X^*\big)^T Q_f \big(X(t_f) - X^*\big) \\ V &= (X(t) - X^*)^T Q (X(t) - X^*) + U^T(t) R U(t) \end{aligned} \quad (3)$$

where $Q_f \ge 0$, $Q \ge 0$ and $R > 0$ are positive weighting matrices. By using the definitions of S and V, the Pontryagin H function can be obtained in the optimal conditions, as follows:

$$\mathcal{H} = V(X(t), X^*, t) + \lambda^T(t)\big(f(X(t), U(t), t)\big) \quad (4)$$

in which $\lambda(t)$ is a quasi-state variable vector with dimensions equal to the state vector $X(t)$. According to the Pontryagin maximum principle, the optimality requirements are:

$$\begin{aligned} X^{\bullet}(t) &= \frac{\partial \mathcal{H}(X^*, X(t), U(t), t)}{\partial \lambda(t)} \\ \lambda^{\bullet}(t) &= -\frac{\partial \mathcal{H}(X^*, X(t), U(t), t)}{\partial X(t)} \\ 0 &= \frac{\partial \mathcal{H}(X^*, X(t), U(t), t)}{\partial U(t)} \end{aligned} \quad (5)$$

and the boundary conditions in these equations are:

$$\begin{aligned} X(t_i) &= X_{t_i} \\ \lambda(t_f) &= \left(\frac{\partial S(X^*, X(t))}{\partial X(t)}\right)_{t_f} = Q_f\big(X(t_f) - X^*\big) \end{aligned} \quad (6)$$

By substituting (1) and (3) into (4), and then applying the necessary optimality requirements of (5), the first-order state-space equations of the system can be found as follows:

$$\begin{aligned} X^{\bullet}(t) &= A\big(X(t_i)\big)X(t) - BR^{-1}B^T\lambda(t) + D(X(t_i), t_i) \\ \lambda^{\bullet}(t) &= -Q(X(t) - X^*) - A^T\big(X(t_i)\big)\lambda(t) \end{aligned} \quad (7)$$

In order to calculate the control signal, (7) can be approximated by the forward Euler method. Then it is as follows:

$$\begin{aligned} \frac{X(t_i+1) - X(t_i)}{h} &= A\big(X(t_i)\big)X(t) - BR^{-1}B^T\lambda(t) + D(X(t_i), t_i) \\ \frac{\lambda(t_i+1) - \lambda(t_i)}{h} &= -Q(X(t) - X^*) - A^T\big(X(t_i)\big)\lambda(t) \end{aligned} \quad (8)$$

that h must be considered small enough to approximate the derivatives well. After simplifying and considering $K_1 = (I + A\big(X(t_i)\big)h)$ and $K_2 = BR^{-1}B^T h$, predicted value for the state vector at the end of the sampling interval is as follows:

$$X(t_i + 1) = K_1 X(t_i) - K_2\lambda(t_i) + hD(X(t_i), t_i) \quad (9)$$

In addition, the value of quasi-state variable vector at the beginning of the sampling interval is as follows:

$$\lambda(t_i) = \big[\big(I + A^T\big(X(t_i)\big)h\big)Q_f + Qh\big](X(t_i + 1) - X^*) \quad (10)$$

where I is an identity matrix. Considering the quasi-state boundary conditions and assuming $t_f = t_i + 1$, the (11) can be rewritten as follows:

$$\lambda(t_i + 1) = Q_f(X(t_i + 1) - X^*) \quad (12)$$

By substituting (33) and (35) into (34), $\lambda(t_i)$ will be:

$$\lambda(t_i) = K_4\big((M^{-1}[K_1 X(t_i) + hD(X(t_i), t_i) + K_2 K_4 X^*]) - X^*\big) \quad (13)$$

in which K_3, K_4 and M are equal to:

$K_3 = \big(I + A^T\big(X(t_i)\big)h\big)$,

$K_4 = \big[\big(I + A^T\big(X(t_i)\big)h\big)Q_f + Qh\big] = \big[K_3 Q_f + Qh\big]$ and

$M = I + K_2 K_4$

Since from (5), it will find that $U(t) = -R^{-1}B^T\lambda(t)$, and by having the value of $\lambda(t_i)$, the control signal will be obtained as follows:

$$U(t_i) = -R^{-1}B^T K_4\big((M^{-1}[K_1 X(t_i) + hD(X(t_i), t_i) + K_2 K_4 X^*]) - X^*\big) \quad (14)$$

If assuming:

$$\begin{aligned} F\big(X(t_i)\big) &= -R^{-1}B^T K_4 M^{-1} K_1 \\ G\big(X(t_i)\big) &= -R^{-1}B^T K_4 M^{-1}(hD(X(t_i), t_i) + K_2 K_4 X^*) + R^{-1}B^T K_4 X^* \end{aligned} \quad (15)$$

then the resulting control signal can be summarized as follows:

$$U(t_i) = F\big(X(t_i)\big)X(t_i) + G\big(X(t_i)\big) \quad (16)$$

By examining (16) it can be observed that the calculation of control signal can be done offline, and therefore, online calculations can be done very fast, because after receiving state feedback and knowing the value of F and G, the value of $U(t_i)$ can be calculated straightforwardly.

Based on the above process, the algorithm of generating control signal can be summarized as follows:

First, obtaining the linear state-space model for the desired system.

Second, defining a performance index based on the purpose of the control system.

Third, finding the Pontryagin H function based on the performance index and system model.

Fourth, attaining the optimality requirements, in accordance with the Pontryagin maximum principle.

Fifth, applying the forward Euler method to derivative approximation for the first-order equations obtained in Step 4.

Sixth, adding boundary conditions to the obtained equations (in this case, it is assumed that $t_f = t_i + 1$ and the terminal error in the next step will go to zero).

Seventh, finding predicted state variable $\left(X(t_i + 1)\right)$ and quasi-state variable vectors$\left(\lambda(t_i)\right)$.

Eighth, achieving offline equations of control signals, based on predicted state and quasi-state vectors.

Suggested Method for Speed and Current Control

A model-based optimal control method for real-time speed regulation of a PMSM is proposed in this section. The goal of this control strategy is to provide an optimal amount of torque that allows driving the PMSM at the desired speed profile, as required by the train control system while satisfying both voltage and current constraints. As presented in Figure 1, the architecture of the proposed control method consists of an outer loop that controls the mechanical dynamics and an inner loop that regulates the electrical components of the machine. The current constraints are checked in the outer loop by limiting maximum torque that can be produced by the machine, and the voltage constraints are considered in the inner loop based on voltage requirements and limitations, for the converter. Therefore, the voltage and current constraints can be separately evaluated and satisfied, and then the performance of the controller is not affected by mutual interferences between them.

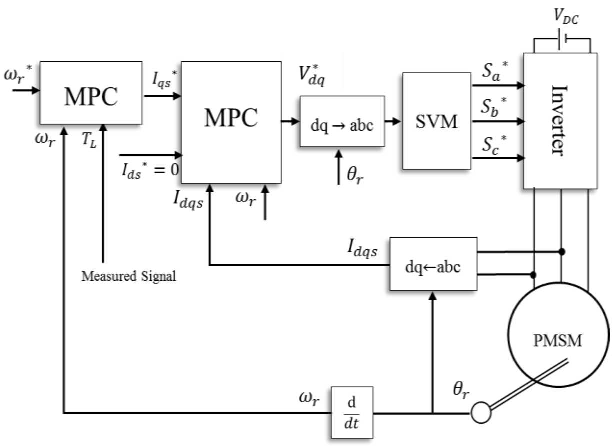

Figure 1. The architecture of the proposed control method

At first, a brief overview of the three-phase PMSM model with a sinusoidal distribution on the stator and a number of permanent magnets on the rotor, in the synchronous rotating frame, is presented. The electrical equations for this machine are:

$$\frac{d}{dt}\begin{bmatrix} i_q^{Mx}(t) \\ i_d^{Mx}(t) \end{bmatrix} = \begin{bmatrix} -\frac{r_s}{L} & -\omega_r^{Mx}(t_i) \\ \omega_r^{Mx}(t_i) & -\frac{r_s}{L} \end{bmatrix} \begin{bmatrix} i_q^{Mx}(t) \\ i_d^{Mx}(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} v_q^{Mx}(t) \\ v_d^{Mx}(t) \end{bmatrix} + \begin{bmatrix} -\frac{\psi_f}{L} \\ 0 \end{bmatrix} \omega_r^{Mx}(t_i) \tag{17}$$

where L_d and L_q are the stator inductances in (H) on the d-axis and the q-axis respectively, r_s is the stator resistance in (Ω), and ψ_f is the magnetic flux produced by rotor magnets in (Wb). These constant values are the same for similar PMSMs. In addition, ω_r^{Mx} is the electrical rotor speed in (rad/s), i_q^{Mx} and i_d^{Mx} are the stator currents in (A) and v_q^{Mx} and v_d^{Mx} are the stator voltages in (V) on the q-axis and the d-axis respectively, for the machine *x* (in a multi-machine system). The mechanical dynamics of each machine is described by the following equations:

$$\begin{aligned} &\frac{d}{dt}\left(\omega_r^{Mx}(t)\right) = \frac{2}{pJ}\left(T_e^{Mx} - T_L^{Mx}\right) - \frac{B_m}{J}\omega_r^{Mx}(t) \\ &\frac{d}{dt}\left(\theta_r^{Mx}(t)\right) = \omega_r^{Mx}(t) \end{aligned} \tag{18}$$

where θ_r^{Mx} is the rotor position in (rad), as well as T_L^{Mx} and T_e^{Mx} are the load and electrical torques respectively, in (N.M), for the machine x. Moreover, *p* is the total number of poles, *J* is the inertia coefficient in (kg.m^2), *B* is the coefficient of viscous friction in (N.M.s), and these constant values are the same for similar PMSMs. The electrical torque T_e^{Mx}, which is consists of the electromagnetic and the reluctance components, can be express as:

$$T_e^{Mx} = \frac{3}{2}\frac{p}{2}\left(\psi_f + \left(L_d - L_q\right)i_d^{Mx}(t)\right) i_q^{Mx}(t) \tag{19}$$

The effects of core saturation and cogging torque are neglected, and it is assumed that the magnets are mounted on the surface of the rotor, therefore $L_d = L_q = L$.

To design current predictive control, the state-space model of (17) is used. Let's rewrite (17) as the following equation:

$$\begin{aligned} &\frac{d}{dt}X_I^{Mx}(t) = A(t_i)\, X_I^{Mx}(t) + B\, U_V^{Mx}(t) + \\ &D\, \omega_r^{Mx}(t_i)\, X_I^{Mx}(t) = \left[i_q^{Mx}(t) \quad i_d^{Mx}(t)\right]^T \\ &U_V^{Mx}(t) = \left[v_q^{Mx}(t) \quad v_d^{Mx}(t)(t)\right]^T \end{aligned} \tag{20}$$

where $X_I^{Mx}(t)$ and $U_V^{Mx}(t)$ are the state vector and the control signals, respectively. The speed variable of $\omega_r^{Mx}(t_i)$ is measured as the state feedback at the time of t_i , and the control signals are designed to drive the machine from its current condition at t_i to the desired condition at $t_{(i+1)}$. During this tiny time interval, ω_r^{Mx} is assumed to be constant, then the control signals are matched in the state-space model of machine *x* in this duration. The feedback controller is intended to force the machine actual currents to track the current references, with the lowest required voltages provided by related converters. Hence, the following performance index is constructed to minimize with respect to (17), as:

$$\begin{aligned} \mathcal{J}_I = \min_{U_V^{Mx}} \Big\{ &\frac{1}{2}\Big[\left(X_I^{Mx}(t_f) - X_I^{Mx^*}\right)^T Q_f \left(X_I^{Mx}(t_f) - \right. \\ &\left. X_I^{Mx^*}\right) + \int_{t_i}^{t_f}\Big[\left(X_I^{Mx}(\tau) - X_I^{Mx^*}\right)^T Q\left(X_I^{Mx}(\tau) - X_I^{Mx^*}\right) + \\ &U^T(\tau)\, R\, U(\tau)\Big] d\tau\Big]\Big\} \qquad \tau \in \left[t_i, t_i + t_f\right] \end{aligned} \tag{21}$$

in which the t_f is the finial time of the prediction horizon, and $X_I^{Mx^*}$ is the desired value of state variables at t_f. The weight matrices of R, Q and Q_f are chosen so as to be diagonal and positive definite.

To find the desired voltage vector of $U_V^{Mx}(t)$, the above optimal problem is solved based on the Pontryagin Maximum Principle as described in the appendix. Then, by applying the necessary optimality condition, first-order state equations are:

$$\frac{d}{dt}\begin{bmatrix} i_q^{Mx}(t) \\ i_d^{Mx}(t) \end{bmatrix} = \begin{bmatrix} -\frac{r_s}{L} & -\omega_r^{Mx}(t_i) \\ \omega_r^{Mx}(t_i) & -\frac{r_s}{L} \end{bmatrix} \begin{bmatrix} i_q^{Mx}(t) \\ i_d^{Mx}(t) \end{bmatrix} - \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} \frac{1}{R_{11}} & 0 \\ 0 & \frac{1}{R_{22}} \end{bmatrix} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \end{bmatrix} + \begin{bmatrix} -\frac{\psi_f}{L}\omega_r^{Mx}(t_i) \\ 0 \end{bmatrix} \tag{22}$$

where the covector $\lambda = [\lambda_1(t) \quad \lambda_2(t)]^T$ can be obtained by the following equation:

$$\frac{d}{dt}\begin{bmatrix}\lambda_1(t)\\ \lambda_2(t)\end{bmatrix} = -\begin{bmatrix}Q_{11} & 0\\ 0 & Q_{22}\end{bmatrix}\left(\begin{bmatrix}i_q^{Mx}(t)\\ i_d^{Mx}(t)\end{bmatrix} - \begin{bmatrix}i_q^{Mx^*}(t)\\ i_d^{Mx^*}(t)\end{bmatrix}\right) - \begin{bmatrix}-\frac{r_s}{L} & \omega_r^{Mx}(t_i)\\ -\omega_r^{Mx}(t_i) & -\frac{r_s}{L}\end{bmatrix}\begin{bmatrix}\lambda_1(t)\\ \lambda_2(t)\end{bmatrix} \quad (23)$$

Afterward, by using the forward Euler discretization in time, the machine voltages as the control signals at every time step of h (where is $h = t_{i+1} - t_i$) are:

$$U_V^{Mx}(t_i) = -R^{-1}B^T K\left(\left(M^{-1}\left[(I + A(t_i)h)X_I^{Mx}(t_i) + hD + (BR^{-1}B^T h)KX_I^{Mx^*}\right]\right) - X_I^{Mx^*}\right) \quad (24)$$

that $K = \left[(I + A^T(t_i)h)Q_f + Qh\right]$ and the matrix M is equal to:

$$M = \begin{bmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{bmatrix}$$

$$m_{11} = 1 + \left(\frac{Q_{11}}{R_{11}} + \left(\frac{1}{h} - \frac{r_s}{L}\right)\frac{Q_{f11}}{R_{11}}\right)\left(\frac{h}{L}\right)^2$$

$$m_{12} = \frac{Q_{f22}}{R_{22}}\left(\frac{h}{L}\right)^2 \omega_r^{Mx}(t_i) \quad (25)$$

$$m_{21} = -\frac{Q_{f11}}{R_{11}}\left(\frac{h}{L}\right)^2 \omega_r^{Mx}(t_i)$$

$$m_{22} = 1 + \left(\frac{Q_{22}}{R_{22}} + \left(\frac{1}{h} - \frac{r_s}{L}\right)\frac{Q_{f22}}{R_{22}}\right)\left(\frac{h}{L}\right)^2$$

To prove the reversibility of matrix M, it is necessary to consider the conditions in which the determinant of M is zero. The determinant of M can be expressed as follows:

$$\mathrm{Det}(M) = \left(1 + \left(\frac{Q_{11}}{R_{11}} + \left(\frac{1}{h} - \frac{r_s}{L}\right)\frac{Q_{f11}}{R_{11}}\right)\left(\frac{h}{L}\right)^2\right)\left(1 + \left(\frac{Q_{22}}{R_{22}} + \left(\frac{1}{h} - \frac{r_s}{L}\right)\frac{Q_{f22}}{R_{22}}\right)\left(\frac{h}{L}\right)^2\right) + \frac{Q_{f11}}{R_{11}}\frac{Q_{f22}}{R_{22}}\left(\frac{h}{L}\right)^4 \omega_r^{Mx^2}(t_i) \quad (26)$$

It is observed that if h is set to be smaller than the electric time constant of the under controlled PMSM, ($h \le \frac{L}{r_s}$), then the determinant of matrix M is always positive. Therefore the matrix reversibility of M can be proved and unique control signals can be achieved. Moreover, if U exceed the predetermined constraint, the rated converter voltage is replaced with a set of calculated command to satisfy the voltage constraints.

Speed Predictive Control

The speed controller regulates the torque component of the current command ($i_q^{Mx^*}(t)$), by evaluating machine acceleration and kinetic energy as control variables. Besides, the flux producing component of the current command ($i_d^{Mx^*}$) is forced to zero in order to maximize the torque density of the traction machine [13]. Any change in the speed command or feedback, during machine acceleration or deceleration, causes electric power oscillations in the drive system. Therefore, to ensure the speed reaches its reference value, while the power fluctuations and speed variations are minimized, a performance index is defined for speed regulator as follows:

$$\mathcal{J}_S = \min_{\omega_r^{\bullet x}}\left\{\frac{1}{2}\int_{t_i}^{t_f}\left[\left(X_S^{Mx^*} - X_S^{Mx}(\tau)\right)^T Q_S(\tau)\left(X_S^{Mx^*} - X_S^{Mx}(\tau)\right)\right]d\tau\right\} \quad (27)$$

wherein this equation, $X_S^{Mx}(\tau) = [\omega_r^{\bullet x}(\tau) \quad E_c^{Mx}(\tau)]^T$, $X_S^{Mx^*} = \left[0 \quad \frac{1}{2}J_t\left(\omega_r^{Mx^*}(t)\right)^2\right]^T$ and $\omega_r^{\bullet x}(\tau) = \frac{d}{dt}\left(\omega_r^{Mx}(\tau)\right)$. In addition E_c^{Mx} is the kinetic energy, $\omega_r^{\bullet x}(\tau)$ is the slope of the angular speed and J_t^{Mx} is the total moment of inertia of machine x (load inertia plus machine inertia). With considering Q_S as a positive definite weighting matrix, and after simplifying (15) it can be described as follows:

$$\mathcal{J}_S = \min_{\omega_r^{\bullet x}}\left\{\frac{1}{2}\int_{t_i}^{t_f}\left[Q_S^{1,1}\left(\omega_r^{\bullet x}(\tau)\right)^2 - \omega_r^{\bullet x}(\tau)\left(Q_S^{1,2}(\tau) + Q_S^{2,1}(\tau)\right)\left(E_c^{Mx^*} - E_c^{Mx}(\tau)\right) + Q_S^{2,2}(\tau)\left(E_c^{Mx^*} - E_c^{Mx}(\tau)\right)^2\right]d\tau\right\} \quad (28)$$

Minimizing this integral cost function with respect to the speed slope of $\omega_r^{\bullet x}$ results in the optimal speed slope as follows:

$$\left(\frac{Q_S^{1,1}}{Q_S^{1,2}(t) + Q_S^{2,1}(t)}\right)\omega_r^{\bullet x}(t) = \frac{2J_t}{p^2}\left(\left(\omega_r^{Mx^*}(t_i)\right)^2 - \left(\omega_r^{Mx}(t_i)\right)^2\right) \quad (29)$$

where $\omega_r^{Mx^*}(t_i)$ and $\omega_r^{Mx}(t_i)$ are the reference and measured angular speeds for machine x, respectively.

If the elements of the weight matrix of Q_S are chosen as $Q_S^{1,2}(t) = \frac{2}{p}\Delta t^2$, $Q_S^{2,1}(t) = \frac{1}{2T_L(t_i)}$ and $Q_S^{2,2}$ being an arbitrary value, then Q_S is a positive definite matrix and (17) can be rewritten as:

$$\frac{2}{p}\omega_r^{\bullet x}(t)T_L(t_i)\Delta t^2 = \frac{2J_t}{p^2}\left(\left(\omega_r^{Mx^*}\right)^2 - \left(\omega_r^{Mx}(t_i)\right)^2\right) \quad (30)$$

By applying the PMSM mechanical equation of (18) and (30) to the electrical torque equation of (19), the torque component of stator current command can be found as follows:

$$i_q^{Mx^*}(t_i) = \frac{8J.J_t^{Mx}}{\Delta t^2.3.p.\psi_f.T_L^{Mx}(t_i)}\left(\left(\omega_r^{Mx^*}\right)^2 - \left(\omega_r^{Mx}(t_i)\right)^2\right) + \frac{8.B_m}{3.p^2.\psi_f}\omega_r^{Mx}(t_i) + \frac{4}{3.p.\psi_f}T_L^{Mx}(t_i) \quad (31)$$

When $i_q^{Mx^*}$ exceed the predetermined constraint, the rated current value is replaced with the calculated command, and therefore the current constraints are satisfied, independently. Consequently, the speed changes only affect the current command, and the voltage constraints are independently limited by the current controller loop.

Torque Estimation Process

The load torque can be estimated by considering the mechanical machine model of (18) and (19) that can be rewritten as follows:

$$T_L^{Mx} = \frac{3}{2}\frac{p}{2}\psi_f.i_q^{Mx}(t) - J\frac{2}{p}\frac{d}{dt}\left(\omega_r^{Mx}(t)\right) - B_m\frac{2}{p}\omega_r^{Mx}(t) \quad (32)$$

Then, using the Backward Euler Approximation for time discretization, (15) can be rewritten as follows:

$$T_L^{Mx}(t_i) = \frac{3}{2}\frac{p}{2}\psi_f.i_q^{Mx}(t_i) - J\frac{2}{p}\left(\frac{\omega_r^{Mx}(t_i) - \omega_r^{Mx}(t_{i-1})}{h}\right) - B_m\frac{2}{p}\omega_r^{Mx}(t) \quad (33)$$

To increase the accuracy of discretization in the implementation, the parameter of T_L^{Mx} is set to the average value of its 10 previous samples as follows:

$$T_L^{Mx}(t_i) = \frac{1}{10}\sum_{z=0}^{10} T_L^{Mx}(t_{i-z}) \quad (34)$$

Impact of Independence Constraints on the Control Process

If a unified MPC strategy has been designed for the proposed drive system, the three state variables of i_d^{Mx}, i_q^{Mx} and ω_r^{Mx} for the PMSM have to be controlled by two control signals of v_d^{Mx} and v_q^{Mx}. In such circumstances, the control signals are highly dependent on the machine speed, and any change in the speed command can cause large variations for both v_d^{Mx} and v_q^{Mx}. Then, when the speed command changes, these control signals are continually confined to the upper and lower bounds by the defined system constraints, and this will cause the controller to deviate from the optimal path. To overcome this issue, most previous studies had suggested that a PI controller, as a part of the cascade control process, determined i_q^{Mx} command [13]. However in this mode, only the error between the reference and the measured speed value reaches zero, regardless of the energy amount required to change the speed, and irrespective of the appropriate speed gradient to produce proper dynamic behavior for the machine. Therefore, the command signals may have unpredictable oscillations and still, the controller cannot provide the optimal path for the state variables.

In described proposed MPC, the command $i_q^{Mx^*}$ are adjusted based on the value of changes in the speed command, and the speed gradient at each sampling interval is determined as a fraction of maximum allowable kinetic energy. In fact, the controller serves the minimum time to reach the final value by optimizing the speed gradient based on the designated cost function. When the calculated speed gradient is so fast that the speed cannot reach the specified value within the very short period of switching time, the nominal current limit will be selected, which means that the controller uses the rated capacity of the machine. Thus, the machine will not be over-powered, and at the same time the speed fluctuations will be eliminated since the limitations of both kinetic energy and winding currents are considered in the control procedure. As a result, no significant fluctuations will be observed, and moreover, when the current command is limited by system constraints, the resulting command will be the best possible way to change the machine speed.

Simulation Studies

To demonstrate the capability of the proposed controller, the circuit models of a traction PMSM and power electronic converters were simulated in the PSIM software, while the control algorithm was managed with the Processor-in-Loop (PIL) Module connected to the TMS320F28335. The specifications of simulated PMSM was obtained from the "XML-SB04A series" datasheet for the PM motors of the LS Company. These specifications are reported in Table 1. The switching method is based on the symmetrical SVM switching pattern. The converters ratings are chosen based on the future test set-up, as shown in Table 2. In the internal and external control loops h parameter is determined as $h = 0.125 \times 10^{-3}$, and the weighting matrices are considered as follows:

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad Q = \begin{bmatrix} 15 & 0 \\ 0 & 85 \end{bmatrix} \quad Q_f = \begin{bmatrix} 280 & 0 \\ 0 & 5800 \end{bmatrix}$$

Table 1. Specifications of XML-SB04A PMSM Series

Machine Parameter	Symbol	Value
Rated Power	P_{rate}	400^{Watt}
Rated Current	I_{rate}	2.89^{A}
Number of Poles	p	8
Stator Resistance per Phase	R_s	0.82^{Ω}
Stator Inductance	$L_d = L_q$	3.66^{mH}
Permanent magnet magnetic flux	ψ_f	0.0734^{wb}
Rated Speed	N_{syn}	$3000^{r.p.m}$
Rated Torque	T_{rate}	$1.27^{N.m}$
Maximum instantaneous torque	T_{max}	$3.82^{N.m}$
Moment of inertia	J	321×10^{-8} $^{Kg.m^2}$
friction coefficient	B_m	0.6×10^{-6} $^{N.m.Sec}$

Table 2. The Rating of Converters

Machine Parameter	Symbol	Value
DC side voltage	$V_{DC} = V_{SC}$	173^{V}
Switching Frequency	F_{SW}	5^{kHz}
On mode Switch Resistance	$R_{DS(on)}$	0.019^{Ω}

The Δt in the speed controller is equal to 0.0118, and the sampling frequency for the feedback signals is 20^{KHz}. The simulation results for suggested MPC algorithm, including speed, torque and currents dynamics for the PMSM are presented in Figure 2. At first, the PMSM is loaded with its rated torque, and its speed is increased from stop mode to the nominal value. Then, at time t_1=0.1S, a 70% step reduction occurred in the load torque. And finally, at time t_2=0.15 S, the speed command returns to 50% of the nominal value and the load torque returns to the nominal value, concurrently and stepwise.

It is observed that machine torque has reached a steady state after 5 milliseconds after start-up. When the torque has a step reduction of 70% at rated speed, output power, appearing on the machine shaft, could lead to severe fluctuations in machine speed. However, by applying proposed MPC, the machine speed only increased by 100 RPM (3.333%), and then it returns to the synchronization at a new optimal rotor angle. Moreover, machine torque has reached the load torque at a short period of time, reducing undesirable machine acceleration. In this case, the controller tries to slow the speed down rapidly by moving the machine to the braking area. However, as the speed approaches the specified reference speed, the controller does not impose any oscillating transient in the presence of increased load, and the control process is well performed by selecting the appropriate control signals at any given moment.

It is obvious that the reduction of the calculated optimal slope is perfectly proportional to the speed rate at which the machine speed approaches its nominal value. Furthermore, the simulation results show that the proposed model-based speed control can provide improved closed-loop performance and can yield a fast response with small overshoots in torque and speed tracking. Therefore, as the suggested control laws are achieved by a simple and fast implementation, the controller has been also minimal in terms of memory allocation during simulation studies.

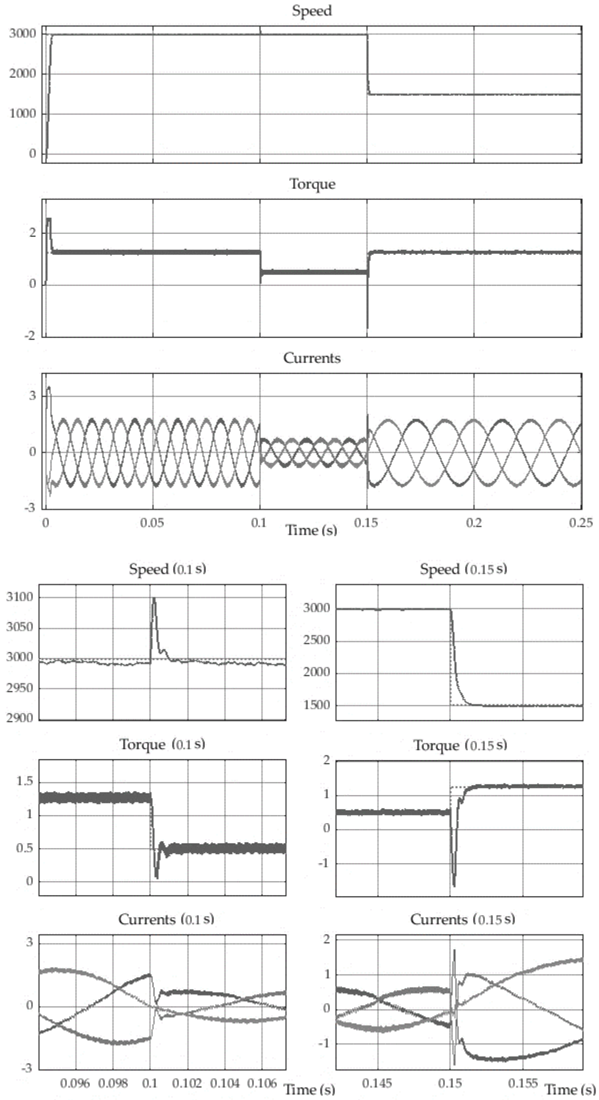

Figure 2. Simulation results: speed, torque and currents dynamics of the PMSM controlled by the proposed MPC method

Conclusions

In this paper, a new model-based optimal control has been developed and implemented on a simulated drive system. To improve the control dynamic performance, the control laws were designed as a set of piecewise affine functions from the control signals based on an offline procedure, and the voltage and current constrain were individually applied through two separate control loops. Therefore, the designed MPC based controller is a constrained method and the control signals can be obtained as a set of linear functions from the feedback states. The mathematical analyses show that the computation time of proposed MPC can be significantly reduced in comparison with the other MPC techniques, providing the possibility of controlling the drive system with a low-cost microcontroller. Moreover, from the simulation results, it is observed that while the controller uses a local linearized system model, it can withstand the non-measurable disturbances.

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