Proposed A Novel Hybrid Meta-Heuristic Algorithm Based on GA, KA and RDA for Solving Integrated Mathematical Model of Cell Formation with Machine Layout and Cell Layout in a Dynamic Environment

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ABSTRACT

Cell Formation (CF) and facility layout design are the two fundamental steps in implementation of the CMS. These decisions are interrelated, therefore addressing them simultaneously is important for a successful design of CMS. In this article, a new non-linear mixed integer programming model is presented which comprehensively considers solving the integrated dynamic cell formation and inter/intra cell layouts in continuous space. In the proposed model, cells are configured in flexible shapes during planning horizon considering cell capacity in each period. This study considers the exact information about facility layout design and material handling cost. To solve the proposed problem as a mixed-integer non-linear programming model is clearly NP-hard, four meta-heuristic algorithms based on an optimization structure are tackled to address the problem. In this regard, not only Genetic Algorithm (GA), Keshtel Algorithm (KA) and Red Deer Algorithm (RDA) are employed to solve the problem, but also a novel hybrid meta-heuristic algorithm based on the benefits of aforementioned algorithms is developed.

Keywords: Cellular Manufacturing System (CMS), Cell Formation (CF), Inter/Intra Cell Layout, Dynamic Cell Formation, Hybrid Meta-heuristic Algorithm.

1. INTRODUCTION

Facility layout is also a key element in designing a CMS which considers the layout of machines within cells (Intra-cell layout) and Layout of cells (Inter-cell Layout) on the shop floor. An efficient facility layout can reduce material handling cost, work-in-process, and throughput rate [1]. A competent layout not only enhances the performance of the system but also minimizes around 40to 50 % of the production costs on average [2]. Although minimizing the number of EEs or other common objectives like the minimization of inter-cell movement cost may reduce the flows between the cells, they do not necessarily lead to a minimum material handling cost, since the real parameters related to the facility layout problem are ignored in the calculation of these objectives. So, incorporating the facility layout problem in the CMS design process is of highly significance. However, layout design in CMS haven't paid much attention, since most of the relevant research only investigate the CFP [3,4]. As stated, facility layout and CF problem decisions are interrelated and tellingly addressing them simultaneously is important for a successful CMS designing [5]. However, each of these decisions is proven to be complex [6,7]. Thus the simultaneous addressing of these decisions is a difficult issue.

Therefore, most of the studies either investigate some of these decisions or handle all, but in a sequential fashion [8]. On the other hand, most approach in the area of facility layout and CF problem, for simplicity, usually consider minimizing the number of inter-cell movements or intra-cell movements or both [9]. Although, for minimizing the material handling cost, the exact information about facility layout design considering the notion of distance must be considered.

Moreover, those approaches that aim at minimizing the material handling cost usually apply unrealistic assumptions such as fixed cells and machines locations in the layout problem. Consequently, the resulting layout may be inefficient. Also, for locating the machines in manufacturing cell space, line formed locations were the only consideration and the machines were assigned to these positions in previous studies. Obviously, if assigning the number of machines to a cell cannot be line formed, it turns into a U- form imposing additional costs to the system.

3. PROPOSED MATHEMATICAL MODEL

The aim of this model is to determine concurrently the formation of cells, the layout of machines inside cells and the layout of cells on the shop floor in dynamic conditions in a way that the total transportation cost of parts and reconfiguration cost of cells and the number of EES are minimized. In the proposed model, the job shop configuration is considered for the intra-cellular layout. The proposed mixed integer nonlinear programming model with a number of assumptions, parameters, and decision variables are discussed below:

3.1. MODEL ASSUMPTION

To simulate the model, the following assumptions are taken into consideration:

- The flow between machines in each period is determined. This number is obtained from the parts demand and parts operational paths as well as batch size of parts transportation.
- The parts are moved within the batches in which the largeness of the batches per product is known and constant for all periods. Also, the size of the part batches is assumed the same for both inter and intra-cell relocations.
- The material handling cost is calculated according to center-to-center distance between machines through a rectilinear distance.
- The material handling cost of inter and intra-cell movements for both parts and machines is related to the distance traveled**.**
- The unit cost of inter and intra-cell movements for each part type is predetermined and remain the same during planning horizon.
- The unit cost of machine relocation during the periods is constant and predetermined for each machine type. This cost includes opening, transferring, and resetting the machine.
- The number of cells to be formed in each period is determined in advance. This predetermined number of cells in the system is on the basis of the expected workload in each cell. However, the shape of the cells in not predetermined and cells are flexibly configured during planning horizon.
- There is only one number of each machine type**.**
- The maximum capacity of cells is known and remains the same during planning horizon.
- Machines are considered as squares of equal area and hence supposed to have a unit dimension. There is no excess inventory between the periods; delayed orders are not allowed and demands per period must be supplied in that period.
- The efficiency of machines and production are assumed 100%.

3.2. SETS

$i,i'=\{1,2,\ldots,,m\}$	Index set of machines
$j=\{1,2,\ldots,,n\}$	Index of parts
$l,k,k'=\{1,2,\ldots,c\}$	Index set of cells
$h=\{1,2,\ldots,,H\}$	Index set for time periods

3.3 MODEL PARAMETERS:

D_{jh}	The demand for part type j in period h
B_j	The largeness of batch for the transportation of part type j
C^j_{intra}	The intra-cell material handling cost for transporting part j per unit distance ($/unit)
C^j_{inter}	The inter-cell material handling cost for transporting part j per unit distance ($/unit)
C_i	The relocation cost of machine *i*($/unit)
R_{ij}	The operation number done on part j using machine i
E	The horizontal length of the shop floor (the length of the shop floor)
F	The vertical length of the job shop (the width of the shop floor)
SP	The set of pairs (*i,j*) such that $a_{ij} \geq 1$ (the set of non-zero elements of part-machine matrix)
NM	The maximum number of machines relocated in each cell per period.
α_j	The coefficient of cost (or penalty) due to the existence of each exceptional part type *j* per period.
N	An appropriate large positive number
$A_{kl}, B_{kl},$	The zero and one random variables
$A_{i i' h}, B_{i i' h}$	The zero and one random variables
$f^j_{ii'h}$	The number of trips for moving part type *j* between machines i and i' in period *h*

$$f^j_{ii'h} = \begin{cases} \left\lceil \dfrac{D_{jh}}{B_j} \right\rceil if\ R_{i'j} - R_{ij} = 1 \\ 0\ if\ R_{i'j} - R_{ij} \neq 1 \end{cases} \tag{1}$$

3.4. DECISION VARIABLES

$$X_{ikh} = \begin{cases} 1 & \text{If machine } I \text{ is assigned to cell } k \text{ in period } h \\ 0 & \text{Otherwise} \end{cases}$$

$$Y_{jkh} = \begin{cases} 1 & \text{If part } j \text{ is assigned to cell } k \text{ in period } h \\ 0 & \text{Otherwise} \end{cases}$$

$$Z_{ih} = \begin{cases} 1 & \text{If machine } i \text{ relocates during periods } h \text{ and } (h+1) \\ 0 & \text{Otherwise} \end{cases}$$

$$U_{ijkh} = \begin{cases} 1 & \text{If} Y_{jkh} = 0 \text{and} X_{ikh} = 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$V_{ijkh} = \begin{cases} 1 & \text{If} Y_{jkh} = 1 \text{and} X_{ikh} = 0 \\ 0 & \text{Otherwise} \end{cases}$$

The horizontal coordinate of the center of machine *i* in period *h*	x_{ih}
The vertical coordinate of the center of machine *i* in period *h*	y_{ih}
The horizontal coordinate ofthe left side of cell *k* in period *h*	p^1_{kh}
The horizontal coordinate of the right side of cell *k* in period *h*	p^2_{kh}
The vertical coordinate of the bottom side of cell *k* in period *h*	q^1_{kh}
The vertical coordinate of the top side of cell *k* in period *h*	q^2_{kh}

Therefore, the relocation cost of part *j* between machines i and i' in period h, regarding inter-cell or intra-cell movement can be determine as follows:

If $X_{ikh}, X_{i'kh} > 0$ this cost equals to Eq. (2) as follow:

$$C^j_{ii'h} = \left(\left| x_{ih} - x_{i'h} \right| + \left| y_{ih} - y_{i'h} \right| \right) C^j_{intra} \tag{2}$$

If $X_{ikh} X_{i'kh} = 0 \wedge X_{ikh} X_{i'k'h} > 0$ this cost equals to Eq.(3) as follow:

$$C^{j}_{ii'h}=\left(\left|x_{ih}-x_{i'h}\right|+\left|y_{ih}-y_{i'h}\right|\right)C^{j}_{inter} \qquad (3)$$

3.1. Mathematical Formulation

With respect to input parameters and variables, the presented nonlinear model for this problem is as follows:

$$Minimize\sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{i'=1}^{m} f^{j}_{ii'h}C^{j}_{ii'h}+\text{¿}\sum_{h=2}^{H}\sum_{i=1}^{m}C_i Z_{ih}+\sum_{h=1}^{H}\sum_{k=1}^{C}\sum_{(i,j)\in sp}\alpha_j.\frac{\left(U_{ijkh}+V_{ijkh}\right)}{2}\text{¿} \qquad (4)$$

$$Subject\text{¿}:$$

$$\sum_{k=1}^{C}X_{ikh}=1,i=1,2,\ldots,m,\forall h \qquad (5)$$

$$\sum_{k=1}^{C}Y_{jkh}=1,j=1,2,\ldots,n,\forall h \qquad (6)$$

$$1\le\sum_{i=1}^{m}X_{ikh}\le NM,k=1,2,\ldots,C,\forall h \qquad (7)$$

$$NZ_{ih}\ge\left|x_{ih}-x_{i(h+1)}\right|+\left|y_{ih}-y_{i(h+1)}\right|\forall i,h<H \qquad (8)$$

$$\left|x_{ih}-x_{i'h}\right|+\left|y_{ih}-y_{i'h}\right|\ge 1 \qquad (9)$$

$$\begin{cases}x_{ih}\ge p^{1}_{kh}-N\left(1-X_{ikh}\right)\\ x_{ih}\le p^{2}_{kh}+N\left(1-X_{ikh}\right)\\ y_{ih}\ge q^{1}_{kh}-N\left(1-X_{ikh}\right)\\ y_{ih}\le q^{2}_{kh}+N\left(1-X_{ikh}\right)\end{cases}\forall i,k,h \qquad (10)$$

$$\begin{cases}p^{1}_{kh}\ge 0\\ q^{1}_{kh}\ge 0\\ p^{2}_{kh}\le E\\ q^{2}_{kh}\le F\end{cases}\forall k,h \qquad (11)$$

$$\begin{cases}p^{1}_{kh}-p^{2}_{lh}+NA_{kl}+NB_{kl}\ge 0\\ p^{2}_{kh}-p^{1}_{lh}-NA_{kl}-N\left(1-B\text{¿¿}kl\right)\le 0\text{¿}q^{1}_{kh}-q^{2}_{lh}+N\left(1-A\text{¿¿}kl\right)+NB_{kl}\ge 0\text{¿}q^{2}_{kh}-q^{1}_{lh}-\end{cases} \qquad (12)$$

In the proposed model, Eq. (9) that prevents machines from being overlapped, can be replaced by the following set of equations due to the unit size of the machines. , ,

$$\begin{cases}x_{ih}-x_{i'h}+NA_{ii'h}+NB_{ii'h}\ge 1\\ x_{i'h}-x_{ih}-NA_{ii'h}-N\left(1-B_{ii'h}\right)\ge 1\\ y_{ih}-y_{i'h}+N\left(1-A_{ii'h}\right)+NB_{ii'h}\ge 1\forall 1\le i<i'\le M\\ y_{i'h}-y_{ih}-N\left(1-A_{ii'h}\right)-N\left(1-B_{ii'h}\right)\ge 1\end{cases} \qquad (13)$$

The first term of the objective function represents the intra- and inter-cellular material transferring costs. The following term denotes the cells reconfiguration cost that may vary from period to period. The third term correlates with decreasing the number of exceptional parts. The coefficient of $\frac{1}{2}$ in this relationship is due to the double calculation of decision variables when they are equal to 1. The first set of constraints (Eq. 5) guarantees that each machine is assigned to only one cell. The second constraint (Eq. 6) ensures that each part is assigned to a single part family. The number of machines in a single cell is limited by Constraint (7).The fourth constraint (Eq. 8) ensures that by relocating machine type i during periods h and (1+h), variable Z_{ih} equals 1. The fifth constraint (Eq. 9) which is replaced with (Eq. 13) prevents machines from being overlapped. As mentioned, the machines are considered as squares with a unit dimension. The set of relationship (10) indicates that each machine must relocate in space of its corresponding cell. The next constraint (Eq. 11) is developed to control the cells which are in space of the job shop. The set of relationship (12) prevents cells from being overlapped.

4. PROPOSED SOLUTION ALGORITHM

Proposed novel hybrid meta-heuristic algorithm (H-RDKGA)

The KA is very good at doing the exploitation action. It seems that the swirling process can be done instead of two processes including roaring and fighting in RDA. Accordingly, for each male, the closest neighbor is specified and the swirling action is done. Due to the mating process, the GA mechanism is considered in this regard. Having a brief illustration, the KA is chosen the intensification properties as well as the GA is measured the diversification phase. This opinion is employed to examine the proposed method with their individual methods and also other feasible alternatives for combinations. Given more details of proposed H-RDKGA, a pseudo-code is provided as seen in Fig. 1.

```
Initialize the Red Deer population.
Calculate the fitness and sort them and form the hinds ($N_{hind}$) and male RDs ($N_{male}$).
Set the Pareto optimal frontier.
while ($t<$ maximum number of iterations)
    for each male RD
        Calculate the distance between this male and all males.
        Select the closest neighbor.
        $S=0$;
        while ($S<$ maximum number of swirling)
            Do the swirling.
            if the fitness of this new position is better than prior
                Update this lucky male.
                break
            endif
            S=S+1
        endwhile
    endfor
    Sort the males and also form the stags and the commanders.
    for each male commander
        Select a hind by roulette wheel selection.
        Mate (crossover) male commander with the selected hind.
    end for
    for each stag
        Select a hind randomly.
        Mate (crossover) stag with the selected hind.
    end for
    Select the next generation via roulette wheel selection.
    Update the Pareto optimal frontier
    t=t+1;
end while
Return the best non-dominated solutions
```

Fig. 1. Pseudo-code of the H-RDKGA

5. COMPUTATIONAL RESULTS

A comparative study is presented in this section. First of all, to enhance the performance of employed metaheuristics and having a fair comparison, a full factorial design method is applied to tune the algorithms' parameters properly. After that, an extensive comparison among meta-heuristics based on different criteria is presented in the following sub-sections.

5.2. COMPARISON AMONG EMPLOYED METAHEURISTICS

This sub-section aims to probe the effectiveness and efficiency of the presented algorithms. Due to it, each meta-heuristic algorithm is performed in all the test problems for 30 times runs. In this case, the behavior of the algorithms in the two objective functions during 30 run times is considered. The behavior of the algorithms in terms of computational time is presented in Fig. 5. As shown in this figure, the behavior of the algorithms is as the same overall. The proposed hybrid algorithm and KA show competitive results in this item. In general, the best algorithm in this criterion is the KA. However, the worst behavior can be concluded from the RDA in most of the testes.

Finally, the average of outputs is saved and utilized to be evaluated by the assessment metrics of Prato-based algorithms. In this regard, Diversification Metric (DM), Spread of Non-dominance Solutions (SNS), Data Evolvement Analysis (DEA) and Percentage of Dominance (POD) are utilized. In all of them, a higher value brings a better capability of algorithms. The details about the evaluation metrics can be referred to some recent studies such as [9,10]. Based on the calculation of these metrics, the outputs of the algorithms for test problems in medium and large sizes are noted in Table 1.

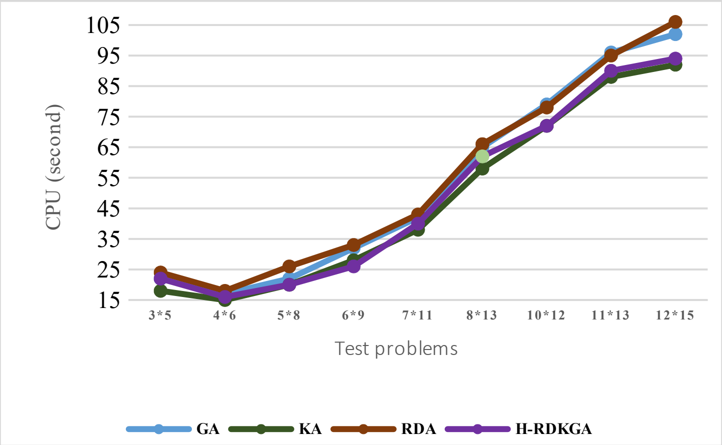

Fig. 2. *Behavior of algorithms in terms of computational time*

Table 1. Evaluation metrics to the performance of the algorithms (i.e., DM, SNS, DEA and POD)

Instances	*DM*				*SNS*				*DEA*				*POD*			
	GA	*KA*	*RDA*	*H-RDKGA*	*GA*	*KA*	*RDA*	*H-RDKGA*	*GA*	*KA*	*RDA*	*H-RDKGA*	*GA*	*KA*	*RDA*	*H-RDKGA*
*3*5*	*14962*	*14389*	*16452*	*16765*	*2498*	*2267*	*1748*	*2699*	*0.18*	*0.16*	*0.12*	*0.15*	*0.16*	*0.22*	*0.14*	*0.22*
*4*6*	*17641*	*17275*	*19743*	*18746*	*6122*	*7210*	*5426*	*7495*	*0.20*	*0.12*	*0.18*	*0.12*	*0.18*	*0.18*	*0.19*	*0.21*
*5*8*	*8124*	*6833*	*7491*	*8945*	*7445*	*7296*	*6948*	*8155*	*0.24*	*0.22*	*0.26*	*0.18*	*0.22*	*0.20*	*0.10*	*0.18*
*6*9*	*34685*	*29164*	*34112*	*35647*	*3485*	*3105*	*2915*	*4039*	*0.28*	*0.14*	*0.22*	*0.14*	*0.15*	*0.14*	*0.11*	*0.16*
*7*11*	*13418*	*12742*	*13671*	*14289*	*2143*	*1834*	*7501*	*2867*	*0.16*	*0.26*	*0.18*	*0.16*	*0.17*	*0.18*	*0.16*	*0.12*
*8*13*	*24914*	*25199*	*23749*	*28763*	*1077*	*1282*	*675*	*2049*	*0.24*	*0.12*	*0.12*	*0.19*	*0.19*	*0.14*	*0.12*	*0.18*
*10*12*	*26493*	*22102*	*25761*	*26714*	*5482*	*4912*	*4466*	*4288*	*0.18*	*0.14*	*0.20*	*0.22*	*0.22*	*0.16*	*0.14*	*0.12*
*11*13*	*31749*	*31054*	*32144*	*33849*	*6388*	*5187*	*5514*	*6382*	*0.26*	*0.18*	*0.14*	*0.18*	*0.22*	*0.18*	*0.14*	*0.16*
*12*15*	*4784*	*7401*	*6195*	*7225*	*6237*	*5853*	*6432*	*7528*	*0.14*	*0.22*	*0.20*	*0.35*	*0.20*	*0.16*	*0.08*	*0.22*

Later, the obtained results for each problem are converted to the Relative Percentage Deviation (*RPD*) computed by:

$$RPD=\frac{\left|Alg_{sol}-Best_{sol}\right|}{Best_{sol}} \tag{14}$$

where Alg_{sol} is the output of algorithm and $Best_{sol}$ is the best value ever found in the problem size. It should be noted that the lower value for the *RPD* is preferred.

6. CONCLUSION

In this paper, a new mixed-integer non-linear programming model was presented to consider the dynamic cell formation and inter/intra-cell layouts in the continuous space simultaneously. The purpose of the model was to determine concurrently the formation of cells and the intra- and inter-cellular layouts in a way that the total transportation cost of parts, the reconfiguration cost of cells, and the number of exceptional elements (EEs) were minimized.

There are several recommendations for future directions of this study. For example, it is interesting to integrate the proposed model with a scheduling problem. The other approach is to use a two-stage or multi-stage stochastic programming method to tackle the uncertainty. From the aspect of the novel proposed hybrid algorithm, more in-depth analyses by other large-scale optimization problems may be considered. At last but not least, new meta-heuristics can be suggested to compare the results of the proposed algorithms.

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