سیزدهمین کنفرانس بین المللی انجمن ایرانی تحقیق در عملیات
۳ و ۴ اردیبهشت ۱۳۹۹
13th International Conference of Iranian Operations Research Society
22-23 April 2020

A problem based on the cost of the location allocation problem and data envelopment analysis

Tahereh Sayyar[1], Jafar Fathaii[2] , Mojtaba Ghiyasi[3]
[1] 1Faculty of Mathematical Sciences, Shahrood University of Technology, Shahrood, Iran; t.sayar.t@gmail.comE-mail
[2] Associate Professor, Faculty of Mathematical Sciences, Shahrood University of Technology, Shahrood, Iran; jf_fathali@yahoo.com
[3] Assistant Professor, Faculty of Management and Industry, Shahrood University of Technology, Shahrood, Iran; mojtaba.ghiasi@gmail.com

ABSTRACT

In this paper, we consider a location allocation problem with considering the data envelopment analysis problem. In this case, the service capacity of each server is considered as its efficiency. Our main goal is to reduce servers to a specified number so that the sum of the traveling cost from customers to servers and the sum of the inputs cost relative to each server (as a DMU) are minimized.

Keywords : Location, Allocation, Data Envelopment Analysis (DEA)

1. INTRODUCTION

Finding the location of facilities in order to serve the clients in an optimal criteria play a fundamental rule in the most models of location theory. The median problem asks to find the location of a facility such that the sum of weighted distances between this facility and the clients are minimized. In the many type of location models the goal is finding the optimal location of facilities with respect to different criteria that consider the time, cost and distances between the clients and facilities. However, the efficiency of the facilities which concerns to the location of facilities, has been considered by a few authors.

In general, term of efficiency can be thought as a type of trade off that what we gain by consuming our resources. Data envelopment analysis (DEA) is a well-known methodology, based on mathematical programming models that can be used to assess the relative efficiency of a number of similar processing units, commonly called decision making units (DMUs). This methodology needs data about the amounts of each input that is consumed by each unit and the amount of each output that is produced by each unit. . In DEA, we seek to find a virtual DMU which produces output equal to the output of the DMU under evaluation, with less input or to produce the output more than output of under evaluation DMU one with same input.

(Thomas et al., 2002) are the first attempts for considering efficiency concept in the location problem. They considered the combination of obnoxious facility location and DEA models and presented two approaches. In the first approach they find the optimal location of facilities, then these optimal facilities are used as the input of the DEA model. If the efficiency of DEA model is unity, then the optimal solution is found. Otherwise, the optimal location of new facilities should be found. This method continues until all facilities are considered or all DEA scores are efficient. In the second approach the DEA model and location problem have been considered as a single objective linear programming model to maximize the efficiency of those facilities that are going to be opened. Then second attempts present by (Klimberg & Ratick, 2008) they used the DEA concept for finding the efficient location of facilities and presented two bi-objective linear programming models for capacitated and un-capacitated facility location models which combined with DEA models. They consider optimization of both spatial interaction between facilities and the customers they serve and the efficiency of facilities at the selected locations simultaneously.

Inspired by (Thomas et al., 2002) and (Klimberg & Ratick, 2008) other research are performed in the literature for combining location problem and efficiency concept. (Guo, 2009) utilized fuzzy DEA for location analysis of finding an ideal location for establishing a Japanese-style rotisserie in a metropolis in China. (Ramanathan, 2009) utilized DEA models for measuring relative attractiveness of locations. He proposed an extension of Huff model capable of dealing with multiple factors using DEA models. The multi-criteria DEA model has been used for location problems in fuzzy environment by (Moheb-Alizadeh et al., 2011). In another study on DEA and location, (Mitropoulos et al., 2013) developed a method for evaluating the efficiency of existing facilities and finding location of the new facilities. In fact, they utilized DEA and location methods separately in two different stages. (Khodaparasti et al., 2016) presented the new formulation combines facility location and DEA to support the decision maker with more realistic solutions based on the optimal location-allocation decisions. In fact, they set the outputs of the model as a function dependent on the allocation variables. (Fang & Li, 2015) proposed a two phase method for incorporating DEA and location. In the first phase they estimated the efficiency of potential location and then in the second phase efficiencmeasures are assumed as a goal in the location problem. The second phase used goal programming model. (Darestani & Mohammadreza, 2016) dealt with a multiple objective programing for local reliability-based maximum expected covering location problem, taking efficiency concept into account that was done by DEA model. (Georgantzinos & Giannikos, 2017) incorporated the efficiency concept to the set covering, packing and partitioning problems. Their attempt yielded to a series of multiple objective programming. In this paper, we propose a new combination of integrating the data envelopment analysis problem into the location problem in which we consider the capacity of the servers according to their efficiency.

In subsection 1.2, we consider a median location problem where the goal is to reduce servers from f to p to minimize total costs. In the subsection 2.2, we discuss the problem of data envelopment analysis and its mathematical model. In Section 3, we discuss the proposed model and develop a peer linearized mathematical model.

2. Preliminary

2.1. Facility location problems

Suppose that there are a set like $N=\{1,2,\dots,n\}$ contains customers and a set $F=\{1,2,\dots,f\}$ contains servers. The cost of setting up jth server is f_j. The cost of traveling of each customer from point $k\in N$ to jth server is c_{kj}. Also suppose each server's service capacity is θ_j. The weight of each demand point is equal to w_k. The goal is to reduce servers from f to p so that minimize the total cost of traveling to the servers and setting up the servers.

$$x_{kj}=\begin{cases}1 \text{ if } kth\ Demand\ is\ allocated\ ¿ & jth\ facility ¿ 0\ otherwise ¿\end{cases}$$

$$e_j=\begin{cases}1 \text{ if } jth\ facility\ remain\ as\ one\ of\ p\ server\\ 0\ otherwise\end{cases}$$

$$Min\sum_{j=1}^{f}\sum_{k=1}^{n} w_k C_{kj} x_{kj}+\sum_{j=1}^{f}\sum_{t=1}^{m} f_j e_j (1)$$

s.t.

$$\sum_{j=1}^{f} e_j = p \quad (2)$$

$$\sum_{j=1}^{p} x_{kj} = 1 \quad k=1,\ldots,n \quad (3)$$

$$\sum_{K=1}^{n} x_{kj} \le \theta_j \quad j=1,\ldots,p \quad (4)$$

$$x_{kj} \le e_j \quad k=1,\ldots,n \quad j=1,\ldots,f \quad (5)$$

$$e_j, x_{kj} \in \{0,1\} \quad j=1,\ldots,f \quad k=1,\ldots,n$$

2.2. DEA Models

Assume there are *n* DMUs which can be banks, schools, ministries, etc. Each DMU uses *m* inputs to produce *s* outputs which may be product service etc. For $j=1,\ldots,n$, let's denote O_{rj}, $r=1,\ldots,m$, outputs of *j*-th DMU and I_{tj}, $r=1,\ldots,s$,inputs of *j*-th DMU. The following CCR model introduced by Charnes et al. (1987) can be used for assessing the efficiency of *k*-th DMU, $k \in \{1,2,\ldots,n\}$.

$$Min\,\theta_k \quad (6)$$

s.t.

$$\sum_{j=1}^{n} \lambda_j I_{tj} \le \theta_k I_{tk} \quad t=1,\ldots,m \quad (7)$$

$$\sum_{j=1}^{n} \lambda_j O_{rj} \ge O_{rk} \quad r=1,\ldots,s \quad (8)$$

$$\lambda_j \ge 0 \quad j=1,\ldots,n$$

The above model is known as envelopment form of CCR model. If we transfer to the dual space we get the following model called multiplier form of CCR model.

3. New location models dealing with efficiency

Consider the n given demand point B_k with nonnegative weight$w_k, k=1,\ldots,n,$. Also suppose that there are f servers which, from the perspective of data envelopment analysis, each server A_j is interpreted as a DMU. Assume that the j-th DMU has a set of m inputs

$I_{j1}, I_{j2}, \ldots, I_{jm}$ and a set of s outputs $O_{j1}, O_{j2}, \ldots, O_{js}$. Consider the capacity of each server proportional to its performance of f units. We want to reduce the number of servers from f to p $(p<f)$ so that sum of the cost of inputs of p selected servers and the sum of weighted cost from this p servers to all demand points is minimized. To model this problem, let

$$e_j = \begin{cases} 1 \; if \; A_j \, is \, selected \\ 0 \; otherwise \end{cases}$$

Then the model of this problem can be written as follows.

$$Min \sum_{j=1}^{f} \sum_{k=1}^{n} w_k C_{kj} x_{kj} + \sum_{j=1}^{f} \sum_{t=1}^{m} c'_{tj} I_{tj} e_j \quad (9)$$

s.t.

$$\sum_{j=1}^{f} e_j = p \quad (10)$$

$$\sum_{j=1}^{p} x_{kj} = 1 \quad k=1,\ldots,n \quad (11)$$

$$\frac{1}{n} \sum_{K=1}^{n} x_{kj} \le \frac{\theta_j}{\sum_{j=1}^{p} \theta_j} \quad j=1,\ldots,p \quad (12)$$

$$x_{kj} \le e_j \quad k=1,\ldots,n \quad j=1,\ldots,f \quad (13)$$

$$e_j, x_{kj} \in \{0,1\} \quad j=1,\ldots,f \quad k=1,\ldots,n$$

4.Conclusion

In this paper, we consider a new combination of integrating the data envelopment analysis problem into the location problem in which we consider the capacity of the servers according to their performance. Then we solved it with the appropriate software and we got the optimal solution.

5. References

[1] Charnes, A., Cooper, W., Rousseau, J. & Semple, J.; ‘‘Data Envelopment Analysis and Axiomatic Notions of Efficiency and Reference Sets”, In: DTIC Document, 1987
[2] Fang, L. & Li, H.; ‘‘Multi-criteria decision analysis for efficient location-allocation problem combining DEA and goal programming”. RAIRO-Operations Research, 49(4), 753-772, 2015.

[3] Georgantzinos, S. & Giannikos, I.; ‘‘A modeling framework for incorporating DEA efficiency into set covering, packing, and partitioning formulations”, International Transactions in Operational Research, 2017.
[4] Guo, P.; ‘‘Fuzzy data envelopment analysis and its application to location problems”, Information Sciences, 179(6), 820-829, 2009.
[5] Khodaparasti, S., Maleki, H., Bruni, M.E., Jahedi, S., Beraldi, P. & Conforti, D.; ‘‘Balancing efficiency and equity in location-allocation models with an application to strategic EMS design”, Optimization Letters, 10(5), 1053-1070, 2016.
[6] Klimberg, R.K. & Ratick, S.J.; ‘‘Modeling data envelopment analysis (DEA) efficient location/allocation decisions”, Computers & Operations Research, 35(2), 457-474, 2008.
[7] Mitropoulos, P., Mitropoulos, I. & Giannikos, I.; ‘‘Combining DEA with location analysis for the effective consolidation of services in the health sector”, Computers & Operations Research, 40(9), 2241-2250, 2013.
[8] Moheb-Alizadeh, H., Rasouli, S. & Tavakkoli-Moghaddam, R.; ‘‘The use of multi-criteria data envelopment analysis (MCDEA) for location–allocation problems in a fuzzy environment”, Expert Systems with Applications, 38(5), 5687-5695, 2011.
[9] Ramanathan, R.; ‘‘Estimating relative attractiveness of locations using data envelopment analysis”, International Journal of Business Performance and Supply Chain Modelling, 1(1), 99-113, 2009.
[10] Thomas, P., Chan, Y., Lehmkuhl, L. & Nixon, W.; ‘‘Obnoxious-facility location and data-envelopment analysis: A combined distance-based formulation”, European journal of operational research, 141(3), 495-514, 2002.