Decentralized Optimal Power Flow in the Presence of Rapid Load Changes

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Abstract

In former times, optimization at distribution and transmission levels was performed in a decentralized manner through a central controller's surveillance. However, given the movement toward microgrid systems and problems of centralized optimization methods, today's systems have approached decentralized structures. On the other hand, privacy protection and information exchange in a safe platform are some of the main concerns and challenges today. Considering the significant development of technology and the smartization of today's power systems, problems, including optimization and resource management, have been considered significantly important within short horizons. This paper examines the decentralized optimal power flow problem within significantly short horizons, a few tens of seconds. The two proposed algorithms are analyzed and compared for problem-solving in terms of solution time and efficiency. Ultimately, the decentralized optimal load flow problem is solved for the PJM network given the rapid load changes and using the Adaptive Alternating Direction Method of Multipliers (A.ADMM) algorithm.

Keywords: Decentralized Optimization, Optimal Power Flow, Quadratic Programming

Introduction

Optimal power flow problem is one of the significant tools for operation and planning topics in power systems studies. Real-time analytics of optimal power flow problems is highly important in large-scale power systems.[1] Hence, the optimal power flow problem is performed by the system operator for operational expense reduction, loss reduction, and voltage profile improvement.[2] The decentralized optimization method is used to reduce computational burden in large-scale problems.[3] Traditional power systems are controlled in a centralized manner using a top-down approach that does not conform to the decentralized optimization method. Therefore, moving away from traditional exploitation to benefit from the advantages of decentralized exploitation is needed.[4] Given the increased distributed energy resources in power systems, decentralized control constantly faces new and extensive agents that complicates the computation of centralized method in large-scale problems. On the other hand, participants in electricity markets are reluctant to share their information to protect their privacy, but possessing all agents' information is needed in decentralized control.[5] Using a decentralized optimization method will not cause any problems and challenges mentioned earlier. Decentralized optimization was previously utilized in multi-agent systems such as transport, radio networks, state estimation, and smart grids.[6] In [7], the authors examined 6 decentralized optimization methods for power flow problems. In [8] The use of the Alternating Direction Method of Multipliers (ADMM) technique to solve the economic dispatch problem has been examined. Article [9] investigates the use of the ADMM method for energy management of multiple linked microgrids in transactive energy (TE) framework and investigates energy management in a decentralized manner in emergency and unwanted circumstances. Article [10] analyses and examines the optimal power flow problem using decentralized optimization methods, and 3 algorithms have been proposed in this article to solve this problem in a decentralized manner.

Given the importance of optimal power flow problems within short periods and the advantages of decentralized optimization methods, this article proposes the decentralized optimization approach with 2 methods for solving optimal power flow problems. Additionally, compared to previous studies, this article evaluates the solution time of decentralized optimal power flow problems within significantly short periods. Therefore, the possibility of optimal real-time exploitation is provided for large-scale systems using a decentralized method.

Optimal Power Flow

The optimal power flow problem is the main base and foundation of planning and exploitation in power systems. [11] DC Optimal Power Flow (DCOPF) seeks to find the optimal distribution of generated resources that will fulfill the network's technical conditions. For 60 years, the optimal power flow problem has been studied extensively and is one of the important challenges of the electric power industry. DCOPF problem is proposed as a linear optimization or quadratic programming. [12] In this article, it is assumed that all generation units are exploited in an online manner, and their binary variables have been surrendered. The mathematical model of the DCOPF problem is given below:

$$F = \arg\min(\sum_i (a_i.pg_i^2 + b_i.pg_i)) \tag{1}$$

$$pg_i - \sum_j \frac{\theta_i - \theta_j}{X_{ij}} = L_i \tag{2}$$

$$p_{ij-min} \le \frac{\theta_i - \theta_j}{X_{ij}} \le p_{ij-max} \tag{3}$$

$$pg_{i-min} \le pg_i \le pg_{i-max} \tag{4}$$

$$\theta_{ref} = \cdot \tag{5}$$

$$|\theta_i| \le \pi \tag{6}$$

Equation (1) optimizes with the goal of minimizing generation cost from the whole network's viewpoint. In the said equation, pg_i is generation power of unit *i* and a_i and b_i are second-order and first-order cost coefficients. Equation (2) points out kcl in each bus. θ_i and θ_j are the voltage angle of bus *i* and the voltage angle of bus *j*, respectively. X_{ij} is the line reactance between bus *i* and *j*, and L_i is the available load in the bus *i*. Constraint (3) shows the limitation of how much power can move through the lines. Constraint (4) points out the generation capacity of each generation unit. Constraint (5) and Constraint (6) present slack bus voltage and angle range of other system buses.

A Review of ADMM

ADMM algorithm is a combination of the dual decomposition algorithm and augmented Lagrangian.[13] Take the following optimization problem into account :

$$\begin{aligned} &\min\ f(x) + f(z) \\ &\quad \text{subject to:} \\ &\qquad Ax+Bz=c \end{aligned} \tag{7}$$

The above objective function is divided into two separate functions of *x* and *z* but connects these two variables' equation constraint. λ is the Lagrangian coefficient that is proportional to the equation constraint of $Ax + Bz = c$. Therefore, Lagrangian equals to:

$$L(x,y,z) = f(x) + g(z) + \lambda^T(Ax + Bz - c) \tag{8}$$

By adding one second-order term to a simple Lagrangian, the augmented Lagrangian can significantly improve the rate of convergence. Augmented Lagrangian is written bellow:

$$\begin{aligned} L_p(x,y,z) = {} & f(x) + g(z) + \lambda^T(Ax + Bz - c) \\ & + \frac{\rho}{2}\|Ax + Bz - c\|_2^2 \end{aligned} \tag{9}$$

ρ is a constant positive coefficient which is called penalty parameter. As said earlier, ADMM is a combination of the dual decomposition algorithm and augmented Lagrangian. ADMM algorithm uses a repetitive process similar to dual decomposition algorithm to update variables of augmented Lagrangian problem. In each iteration *k*, ADMM consists of the below steps

$$x^{k+1} = \arg\min L_p(x, z^k, \lambda^k) \tag{10}$$

$$z^{k+1} = \arg\min(L_p(x^{k+1}, z, \lambda^k)) \tag{11}$$

$$\lambda^{k+1} = \lambda^k + \rho(Ax^{k+1} + Bz^{k+1} - c) \tag{12}$$

In the first step, variable vectors of z^k and λ^k, which were achieved in the previous iteration of *k*, are considered constant, based on which augmented Lagrangian is solved for the variable vector of *x*. Hence, updated x^{k+1} is achieved. Next, the same process is applied to the variable vector of *z*. Ultimately in the third step, Lagrangian coefficients corresponding to constraint $Ax + Bz = c$, which connects variables of *x* and *z* together, are updated given the extent of convergence from this constraint and penalty parameter.

The termination criterion of the ADMM algorithm is calculated by defining two parameters, including primal residual and dual residual.

$$r^k = \|Ax + Bz - c\| \tag{13}$$

$$s^k = \rho\|A^T B\ (z^k - z^{k-1})\| \tag{14}$$

When these parameters are less than a predetermined value, the ADMM algorithm will be terminated. Figure 1 presents the graphical illustration of this optimization method.

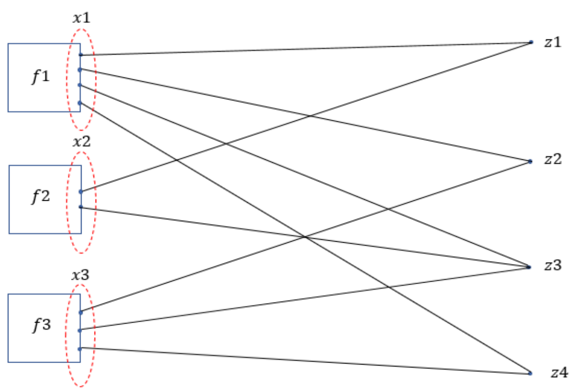

Fig 1. Graphical illustration of decentralized optimization method

Decentralized Optimal Power Flow

Problem modeling of optimal power flow of DCOPF is given below using ADMM algorithm.

$$F_i = \min\left\{ C_i(pg) + (\lambda_i^k)^T(\theta_i - z^k) + \frac{\rho}{2}\left\|\theta_i - z^k\right\|_2^2 \right\} \quad (15)$$

Subject to:

$$C_i(pg_i) = a_i pg_i^{\ 2} + b_i pg_i \quad (16)$$

$$z^{k+1} = \frac{1}{N}\sum_{i=1}^{N}\theta_i^{k+1} \quad (17)$$

$$\lambda_i^{k+1} = \lambda_i^k + \rho(\theta_i^{k+1} - z^{k+1}) \quad (18)$$

$$\left\|\lambda_i^{k+1} - \lambda_i^k\right\|_2^2 \le \varepsilon \quad (19)$$

$$\rho\left\|\theta_i^{k+1} - \theta_i^k\right\|_2^2 \le \varepsilon \quad (20)$$

Given previously-said facts, the initial DCOPF problem, which was controlled in a central form, is decomposed into local subproblems that variables θ and Z are named local variable and global variable, respectively.

As an example, Figure 2 shows the modeling and information exchange in a two-bus system.

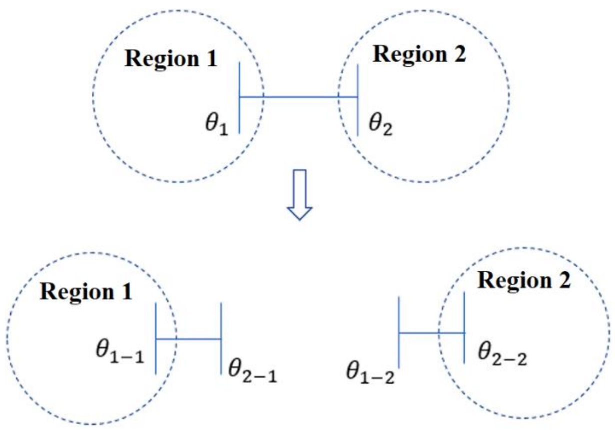

Fig2. Information exchange in a two-bus system

In Figure (2), it is obvious that two regions are connected using one line. To virtually separate these two regions, region 1 and region 2 in DCOPF problem only need to share information related to the angle of marginal buses with each other, and there is no need to share private information, e.g., power generation limitation, power generation cost function, amount of load, and etc. Upon modeling the problem as above and using the ADMM optimization technique, the centralized optimal power flow problem can be converted into a decentralized optimal power flow problem.

Calculating the proper value of ρ is highly effective in the convergence rate of the ADMM algorithm, and this coefficient depends on the proposed optimization model. This article compares 2 methods in 2 circumstances: constant and variable ρ. If ρ is considered a variable, an initial value is first given ,ρ and then the value of ρ is updated in each iteration using Equation (21). Next, it is shown that the method with variable ρ converges to the final solution in lesser iterations and a shorter time than the circumstance in which ρ is considered constant.

$$\rho^{k+1} = \begin{cases} \xi\rho^k & \text{if } \left\|r^k\right\|_2^2 > \gamma\left\|s^k\right\|_2^2 \\ \dfrac{\rho^k}{\xi} & \text{if } \left\|s^k\right\|_2^2 > \gamma\left\|r^k\right\|_2^2 \\ \rho^k & \text{otherwise} \end{cases} \quad (21)$$

In this article, values of ξ and ϒ are assumed to be 2 and 10.

Network Under Investigation

Figure (3) shows a schematic diagram of the PJM network with 5 buses and 6 lines. In this network, it has been assumed that 3MW load, 3 MW load, and 4MW load have been placed in bus B, bus C, and bus D, respectively. In this article, the PJM network is divided into 3 subsystems, similar to Figure 3. Data related to generation lines and units have been used from the article [14].

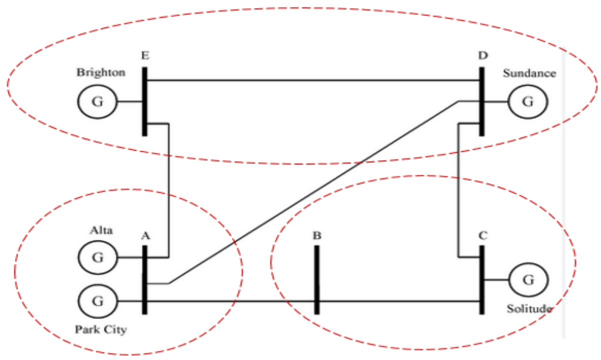

Fig 3. PJM network

Table 1 compares the results achieved using centralized and decentralized optimization methods. The results depict that generators’ generation power in decentralized and centralized methods is close to each other with highly significant precision.

Table1. Comparing generators’ generation power in centralized and decentralized circumstances

Generator	Centralized generated power (MW)	ADMM generated power (MW)	A.ADMM generated power (MW)
pg_1	0.4	0.4	0.4
pg_2	1.7	1.7	1.7
pg_3	1.9	1.8998	1.8997
pg_4	0	0	0
pg_5	6	6	6

Table 2 compares the 2 proposed algorithms in terms of iteration and solution time. The results depict that the Adaptive Alternating Direction Method of Multipliers (A.ADMM) algorithm, in which ρ is considered a variable, converges to the final solution in lesser iterations and a shorter time than the classic ADMM algorithm with a constant ρ. This advantage makes the method highly applicable in studies where solution time is an important and determinant matter.

Table 2. Comparing ADMM and A.ADMM methods

ρ	Number of iterations ADMM	Solution time (s) ADMM	Number of iterations A.ADMM	Solution time (s) A.ADMM
40	14365	281.1492	1170	17.877
60	9420	179.604	924	16.03
80	6686	131.186	990	17.32
10	5157	95.509	896	15.588
12	6283	110.165	958	17.03

Next, this article investigated how the generation power of units and voltage angle of marginal buses converge.

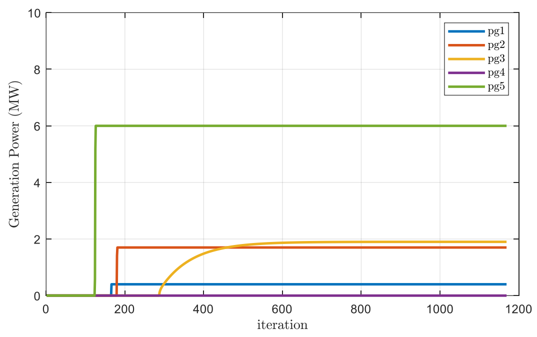

Fig 4. Generators’ generation power in A.ADMM method

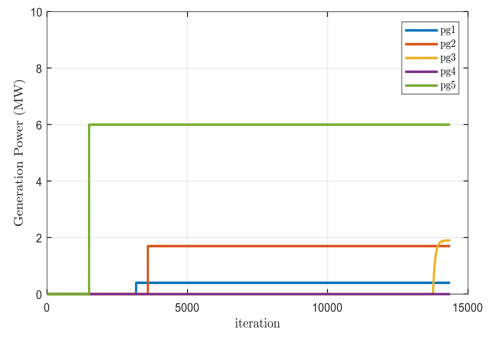

Fig5. Generators’ generation power in ADMM method

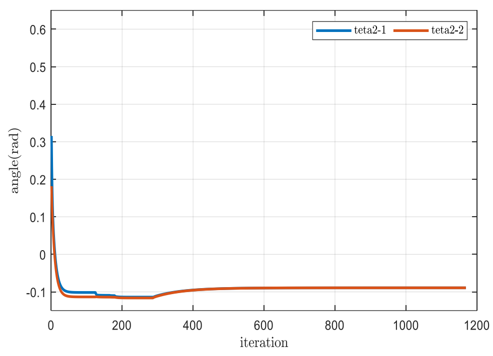

Fig 6. Angle of marginal bus B in A.ADMM method

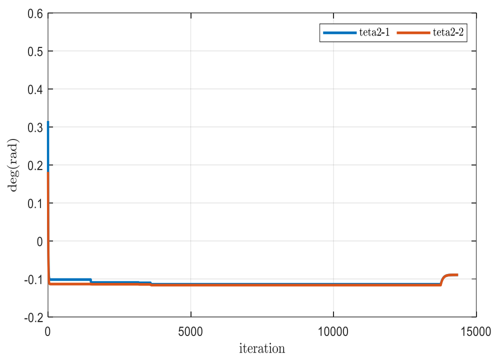

Fig7. Angle of marginal bus B in ADMM method

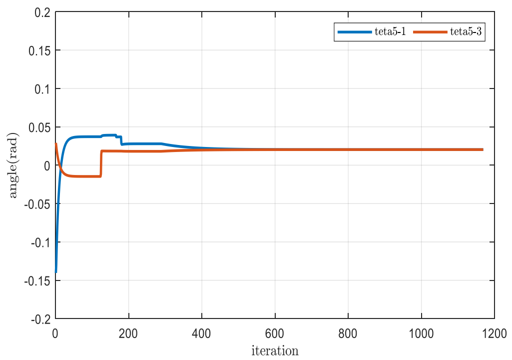

Fig 8. Angle of marginal bus E in A.ADMM method

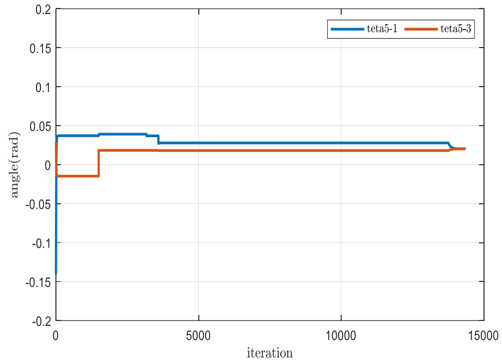

Fig 9. Angle of marginal bus E in ADMM method

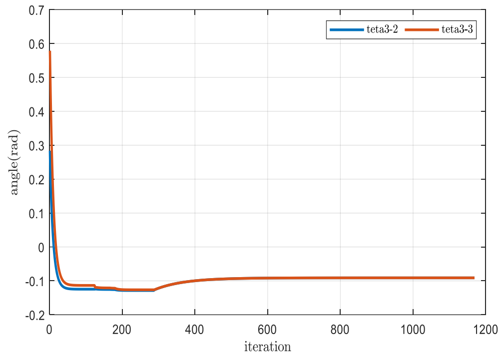

Fig 10. Angle of marginal bus C in A.ADMM method

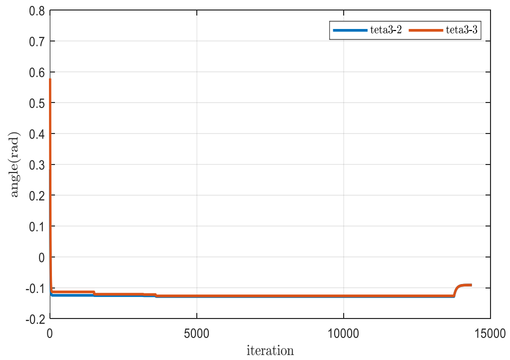

Fig 11. Angle of marginal bus C in ADMM method

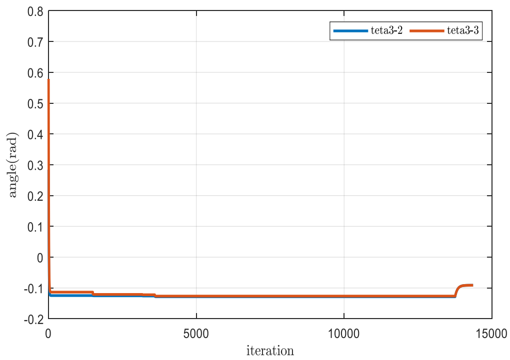

Fig 12. Angle of marginal bus D in A.ADMM method

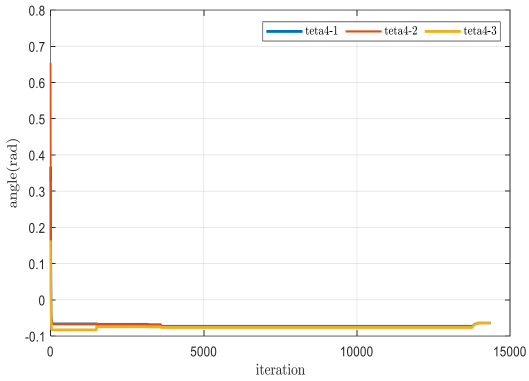

Fig 13. Angle of marginal bus D in ADMM method

Figure 14 shows load changes every 30 seconds for a specific hour, and due to A.ADMM method's efficiency in problem-solving within significant short horizons, the said technique is utilized to solve DCOPF problem in a time frame of 30 seconds. In this condition, each generator's generation power is depicted in Figure 15.

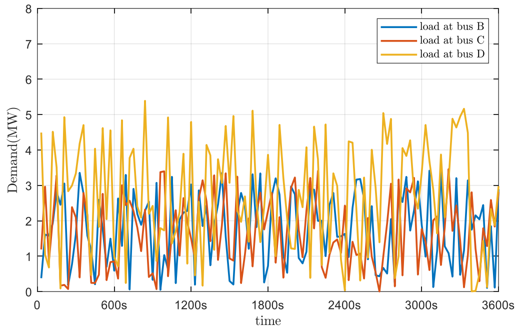

Fig 14. Rapid changes in available load

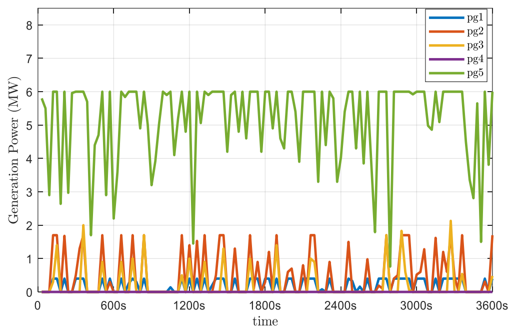

Fig 15. Distribution of generated power when load is rapidly changing

Conclusion

This article investigates optimal power flow in a decentralized form. Moreover, the two decentralized optimization techniques are analyzed and compared for problem-solving in terms of solution time. As said in previous sections, the solution time of optimization problems is significantly important in exploitation and energy management studies due to today's advancement of technology. It was shown that exploitation of power systems within significantly short periods and making decisions in a faster form using A.ADMM is possible compared to the classic ADMM technique. Moreover, compared to previous studies, the decentralized optimal power flow problem has been analyzed within a time frame of 30 seconds in this study, the results of which are reported. To continue in the same direction, we suggest decentralized energy management in microgrids and energy exchange in significantly short times in farmwork of transactive energy using the proposed techniques.

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