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Hybrid Completely Positive Tensorial PCA method for face recognition

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Abstract. Tensors are powerful tools for compressing data in any format. Tensorail methods such as tensor completion have many applications in various fields of image processing, artificial intelligence and data science. Nowadays, tensorial methods have numerous applications in data science. On the other hand, principal component analysis (PCA) as a statistical dimensionality reduction method has different applications for image processing. In this paper, we discuss hybrid methods between tensors and PCA for face recognition. Based on our experiments, this method has high efficiency and computational cost.
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1. Introduction

Matrix and tensor completion methods have many applications in various fields of big data analysis, prediction based on collected data, image processing, and computer vision. Incomplete, distorted, and noisy data has always been a major challenge in the field of big data analysis, especially image processing [5]. This problem appears in digital images as a variety of noise and image distortion. In the face detection, these problems are very importatnt, matrix and tensor completion methods have the ability to compensate to a significant degree (up to 90 percent distortion) [10]. On the other hand, dimentionality reduction methods, for example PCA have the ability to reduce the dimension and eliminate noise and outliers data, significantly increase computational efficiency [6]. The human face is a primary focus of attention in society and playing an important role in conveying identity and emotion. Face recognition is a technology which recognizes a person by his/her face image [3]. Different and numerous methods have been proposed for this work. The problem of human face recognition is complex, because it has to deal with a variety of parameters, including illumination, pose orientation, expression, head size, image obscuring and face bachground [3]. The use of hybrid methods has become very common today. By combining these methods from statistical learning and tensor algebra, advanced and precise methods can be achieved that, while having high efficiency with significant detection and recovery rates, also have high computational efficiency [7].

2. Notations and Preliminaries

In this section, we briefly state some preliminaries for PCA, tensor calculus and tensor completion. For more details and information, please refer to [5], and [10].

2.1. Definition. A tensor is a multidimensional array, The dimensionality of it is described as its order. An Nth-order tensor is an N-way array, also known as N-dimensional or N-mode tensor, denoted by X. We use the term order to refer to the dimensionality of a tensor (e.g., Nth-order tensor), and the term mode to describe operations on a specific dimension (e.g., mode-n product) [2]. We denote the set of all n-dimensional tensors of order m by $T_{m,n}$. For a tensor A, if all of $a_{i_1,\dots,i_n}$ are invariant under any permutation of indices, then A is called a symmetric tensor. We show the set of all real n-dimensional symmetric tensors of order m with $S_{m,n}$.

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Tensors are simply mathematical objects that can be used to describe physical properties. In fact, tensors are merely a generalization of scalars, vectors and matrices; a scalar is a zero rank tensor, a vector is a first rank tensor and a matrix is the second rank tensor [10].

2.2. Definition. The inner product of two tensors X and Y of the same size is defined as $< X, Y >$. Unless otherwise specified, we treat it as dot product defined as follows [9]:

$$< X, Y > := \sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \cdots \sum_{i_N=1}^{I_N} X_{i_1, i_2, \cdots, i_N} y_{i_1, i_2, \cdots, i_N} \tag{1}$$

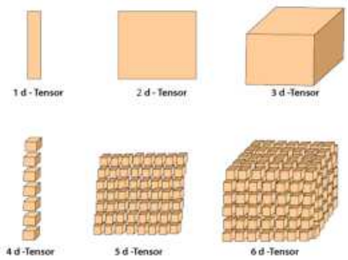

Figure 1. The representation of tensors as a n-way array.

2.3. Definition. Generalized from matrix Frobenius norm, the F-norm of a tensor X is defined as [5]:

$$||X||_F := \sqrt{< X, X >} = \sqrt{\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \cdots \sum_{i_N=1}^{I_N} X^2_{i_1, i_2, \cdots, i_N}} \tag{2}$$

2.4. Definition. Suppose X is a symmetric tensor from $S_{m,n}$, r is a positive integer number and $u^{(k)} \in \mathbb{R}^n$ for $k \in \{1, \cdots, r\}$ are exist such that

$$X = \sum_{k=1}^{r} (u^{(k)})^m \tag{3}$$

Therefore, X is called a completely positive tensor (CP), and (3) is CP-decomposition of X (For example, see Figure 2). In the CP-decomposition (3), the minimum of r is called CP-rank of X [5].

2.5. Definition. The well-known optimization problem for matrix completion as follows:

$$\begin{aligned} & Min_X : \frac{1}{2}||X - M||^2_\Omega \\ & s.t. rank(X) \leq r, \end{aligned}$$

where $X, M \in \mathbb{R}^{p \times q}$, and the elements of M in the set Ω are geven while the remaining are missing. We aim to use a low rank matrix X to approximate the missing elements [1].

2.6. Definition. The tensors is the generalization of the matrix concept. Given a low-rank (either CP rank or other ranks) tensor T with missing entries, the goal of completing it can be formulated as the following optimization problem [10]:

$$\begin{aligned} & Min_X : \frac{1}{2}||X - Y||^2_F \\ & s.t. ||X||_{tr} \leq c \\ & Y_\Omega = T_\Omega \end{aligned}$$

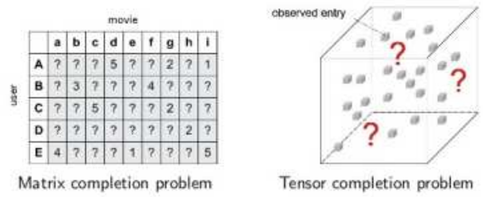

FIGURE 2. The comparision scheme of matrix vs tensor completions.

where X, Y, T are n-mode tensors with identical size in each mode. For this paper, we propose the following definition for tensor trace norm based on the completed positive (CP) rank as follows [2]:

$$||X||_{tr} = \frac{1}{n}\sum_{i=1}^{n}||X_i||_{tr} \tag{4}$$

In essence, the trace norm of a tensor is the average of the trace norms of all matrices unfolded along each mode.

Figure 2 shows the The comparision between matrix and tensor completion problems.

2.7. DEFINITION. PCA is a statistical procedure that uses orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. PCA produces the optimal linear least-squares decomposition of a training set [6]. In computing the PCA representation, each image is interpreted as a point in $\mathbb{R}^{n\times m}$, where each image is n by m pixels. PCA finds the optimal linear least-squares representation in $(N-1)$-dimensional space, with the representation variance [3]. The PCA representation is characterized by a set of $N-1$ eigenvectors and eigenvalues. The orthonormal basis of these eigenvectors are called principal components [6].

Figure 3 shows the The scheme of PCA method for a data set of genes.

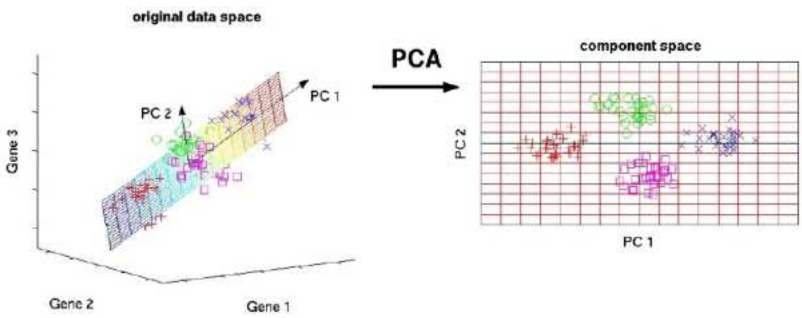

FIGURE 3. The scheme of PCA method.

3. Main results

Real-world data, such as speech signals, digital photographs, or MRI scans, usually have high dimensionality. In order to handle such real-world data adequately, its dimensionality needs to be reduced. Dimensionality reduction is the transformation of high-dimensional data into a meaningful representation of reduced dimensionality [4]. Ideally, the reduced representation should have a dimensionality that corresponds to the intrinsic dimensionality of the data. The intrinsic dimensionality of data is the minimum number of parameters needed to account for the observed properties of the data [7]. As a result, dimensionality reduction facilitates among others, classification, visualization, and compression of high-dimensional data. Traditionally, dimensionality reduction was performed using linear techniques such as Principal Components Analysis (PCA), factor analysis, classical scaling, and t-SNE [4]. This method is a combination of principal component analysis (PCA) and CP tensor analysis. We implemented our codes, algorithms, and experiments on the Asus Laptop with CPU Ci7-2670QM (2.2-3.1 GHz), RAM 16GB (DDR3), GPU Nvidia Geforce GT 540M (2GB), and OS Win-10 (64bit) in the $MATLABR2021a$. The images are first inserted and then tensorized, for example, each image can be a $200 \times 200 \times 3$ tensor, which means that the image has a size of 200 by 200 pixels and has three RGB color layers (R: Red, G: Green, B: Blue). In the next step, the tensor factorization operation of CP type is performed. By focusing on the image tensors, the low-rank approximation is performed, and by linear search, the most appropriate and closest image to the main image is identified and displayed. This method is at a good level in terms of memory management and computational efficiency [6]. In practice, a montage of images of tensor components is produced. Here a set of 40 images are compared. Figure 4

FIGURE 4. The main data set of test images.

show the main data set of test images. After import this image data set, the algorithm tensorized images in the RGB panel and produced the images shown in figure 5. After that, the face images are reconstructed from the tensorized images produced in the previous steps, two sets of images are displayed, one is the original images and the other is the reconstructed images. In the figure 6, you can see the compatibility and high accuracy of these two categories of images. According to studies

FIGURE 5. The tensorized images in the RGB panel.

conducted in our experiments with 1000 iterations and stochastical initialization, in addition to computational savings due to reduced dimensions by PCA, this method has high detection and recognition rates of more than 90 percent in some cases (exactly 91.27 percent) and error is about 10^{-6}. On the other hand, this method has suitable computational costs and more efficient than other methods.

4. Conclusions

In this paper, we have introduced a new computational hybrid method based on the tensors and PCA, that lead to advanced hybrid algorithms for the registration, detection, and recognition of

FIGURE 6. The reconstructed images.

human faces. The combination of conventional linear dimensional reduction methods such as PCA and LDA with tensors and the development of new algorithms such as MPCA and MLDA is a testament to this claim. In any type of problem, depending on the case study conditions such as type of images or data (structured, semi-structured, or unstructured), by choosing the appropriate tensor analysis method, multiplication and metric, the type of optimization method depending on equal or unequal constraints of the problem, or convexity or concavity, the best method, and algorithm for achieved a better result, should be selected. Finally, the hybrid between tensors and dimensionality reduction methods (like PCA) results in efficient and hopeful methods for big data analysis, especially digital images.

References

[1] E. J. Candes, and B. Recht, *Exact Matrix Completion via Convex Optimization*, Found Comput Math, no.9, 717 (2009), Springer.

[2] M. Kurucz, A. Benczur, and K. Csalogany, *Methods for large scale svd with missing values*, KDD Cup (2007), ACM 978-1-59593-834-3/07/0008.

[3] P. Kamencay, R. Hudec, M. Benco, and M. Zachariasova, *2D-3D Face Recognition Method Based on a Modified CCA-PCA Algorithm*, International Journal of Advanced Robotics Systems (2014).

[4] J.M. Lewis, L.J.P. van der Maaten, and V.R. de Sa. *A Behavioral Investigation of Dimensionality Reduction*, In Proceedings of the Cognitive Science Society (CSS) (2012), 671-676.

[5] J. Liu, P. Musialski , P. Wonka, and Ye J., *Tensor Completion for Estimating Missing Values in Visual Data*, IEEE Transactions on Pattern Analysis and Machine Intelligence (2013), no. 1 (35), 208-220.

[6] H. Moon, and P. J. Philips, *Computational and performance aspects of PCA-based face-recognition algorithms*, Perception (2001), no. 30, 303-321.

[7] L. J. P. Van der Maaten, EO Postma, HJ Van den Herik, *Dimensionality reduction: A comparative review*, Technical Report TiCC TR (2009), no. 5.

[8] L. Qi, *The Spectral Theory of Tensors*, arXiv: 1201.3424v1, (2012).

[9] A. Rovi, *Analysis of 2-Tensors*, MAI mathematics: Master thesis (2010), Linkopings University.

[10] Q. Song, H. Ge, J. Caverlee and X. Hu, *Tensor Completion Algorithms in Big Data Analytics*, ACM Transactions on Knowledge Discovery from Data (TKDD) (2019), no. 6 (13), 148.

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