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Fuzzy nominal sets

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Abstract. In this paper we fuzzify the notion of nominal sets by universal algebraic approach. We introduce the category of fuzzy nominal sets and consider its relation to other familiar categories. Also we define fuzzy freshness relation and present some results about it.
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1. Introduction

The notion of a nominal set was introduced by Fraenkel in 1922 [4], and developed by Gabby and Pitts [5, 9]. Since then, nominal set theory have become a live topic in semantics and a subject of interest of computer scientists. The concept of the actions of a fuzzy semigroup on a fuzzy set is presented in [6]. In [6, Definition 3.2] there are two approaches to introduce the fuzzy S-act, classic and universal algebraic. Here we develop the notion of nominal sets by universal algebraic fuzzification of nominal sets and introduce the category of fuzzy nominal sets in (Definition 3.2) and study its relation to the categories **Nom** and **FSet**. Finally, we define fuzzy freshness relation in (Definition 5.3).

2. Preliminaries

In this section, we recall some definitions and results that will be effectively employed in this paper.

Suppose $\mathbb{D}$ is a fixed (countable) infinite set. A permutation π of $\mathbb{D}$ is a bijection map from $\mathbb{D}$ to itself. The permutations of $\mathbb{D}$ with composition and identity form a group, called the *symmetric group* on the set $\mathbb{D}$ and denoted by Sym$\mathbb{D}$. A permutation $\pi \in \mathrm{Sym}\mathbb{D}$ is finitary if the set $\{d \in \mathbb{D} \,|\, \pi d \neq d\}$ is a finite subset of $\mathbb{D}$. The subgroup of Sym$\mathbb{D}$ of finitary permutations, denoted by Perm($\mathbb{D}$). The pair $(X, \cdot)$ is a Perm($\mathbb{D}$)-set if X is a set and $\cdot$ is the action map $\mathrm{Perm}(\mathbb{D}) \times X \to X$ mapping (π, x) to πx such that for every $\pi_1, \pi_2 \in \mathrm{Perm}(\mathbb{D})$ and every $x \in X$ we have $\pi_1(\pi_2 x) = (\pi_1 \circ \pi_2)x$ and $idx = x$.

Perm($\mathbb{D}$)-sets are the objects of a category, denoted by Perm($\mathbb{D}$)-**Set** whose morphisms are action preserving maps and compositions and identities are as in the category **Set** of sets and maps.

2.1. Category of nominal sets.

In this subsection, we give some basic facts about nominal sets. For more information see [3, 9].

2.1. Definition. (i) Let X be a Perm($\mathbb{D}$)-set and $x \in X$. Then, a finite set $S \subseteq \mathbb{D}$ supports x if and only if, for all $\pi \in \mathrm{Perm}(\mathbb{D})$ and for every $d \in S$, we have $(\pi(d) = d \Longrightarrow \pi x = x)$.

(ii) A Perm($\mathbb{D}$)-set X is a *nominal set* if all elements of X have finite supports.

(iii) An *equivariant map* from a nominal set X to a nominal set Y is a map $f : X \to Y$ with $f(\pi x) = \pi f(x)$, for all $x \in X, \pi \in \mathrm{Perm}(\mathbb{D})$.

Nominal sets are the objects of a category, denoted by **Nom**, whose morphisms are equivariant maps and whose composition and identities are as in the category of Perm($\mathbb{D}$)-**Set**.
(iv) A subset $Y \subseteq X$ is called *equivariant subset* if it is closed under group action. More explicitly, for every $\pi \in$ Perm($\mathbb{D}$) and $y \in Y$ we have $\pi y \in Y$.

Notation. The category **Nom** is a full subcategory of the category Perm($\mathbb{D}$)-**Set**.

2.2. Remark. Suppose X is a nominal set and $x \in X$. Intersection of two supports of x is a support of x. So x has the least support and denoted by $\operatorname{supp} x$. In fact, $\operatorname{supp} x = \bigcap\{C : C \text{ is a finite support of } x\}$.

2.3. Definition. An element x of a nominal set X is called a *zero* element if $\pi x = x$, for every $\pi \in$ Perm($\mathbb{D}$). We denote the set of all zero elements of a nominal set X by $\mathcal{Z}(X)$. The nominal set of all zero elements of whose elements are zero is called a *discrete* nominal set, or a nominal set with the *identity action*.

2.4. Example. (i) The set $\mathbb{D}$ with the action $\cdot : \mathrm{Perm}(\mathbb{D}) \times \mathbb{D} \to \mathbb{D}$ where mapping $(\pi, d) \rightsquigarrow \pi d$ is a Perm($\mathbb{D}$)-set and each $d \in \mathbb{D}$ is finitely supported by $\{d\}$ and this is the smallest support, so that $\operatorname{supp} d = \{d\}$.
(ii) Let $\pi \in$ Perm($\mathbb{D}$). Then, Perm($\mathbb{D}$) with $\pi_1 \rightsquigarrow \pi^{-1}\pi_1\pi$ is a nominal set. Indeed, $\operatorname{supp} \pi = \{d \in \mathbb{D} : \pi d \neq d\}$.
(iii) The discrete nominal set on a set X is given by the Perm($\mathbb{D}$)-action $\pi x = x$, for $x \in X$ and $\pi \in$ Perm($\mathbb{D}$). Then we have $\operatorname{supp} x = \emptyset$, for each $x \in X$.

2.5. Definition. For any nominal set X, the map $\operatorname{supp}_X : X \to \mathcal{P}_{\mathrm{f}}(\mathbb{D})$ sending each x to the finite set $\operatorname{supp} x$ is called *support map*. The support map is an equivariant map.

Notation.(i) Suppose X is a nominal set. The Perm($\mathbb{D}$)-orbit of an element $x \in X$ is $\mathrm{orb}x = \{\pi x : \pi \in \mathrm{Perm}(\mathbb{D})\}$.
(ii) For a nominal set X and $x, x' \in X$, the relation $\sim$ defined by $(x \sim x' \Leftrightarrow (\exists \pi \in \mathrm{Perm}(\mathbb{D})), \pi x = x')$ is an equivalence relation. The quotient set $X/\sim$ is usually written as $X/\mathrm{Perm}(\mathbb{D}) = \{\mathrm{orb}x : x \in X\}$.

2.2. Fuzzy Perm($\mathbb{D}$)-set.

Here we briefly recall relevant definitions concerning fuzzy Perm($\mathbb{D}$)-sets. For the most part we follow [6].
A fuzzy set is a set X together with a map $\mu_X : X \to [0,1]$ is called a *fuzzy set* (over X), denoted by (X, μ_X). The set of X is called the underlying set and μ_X is called the membership map of the fuzzy set (X, μ_X), and $\mu_X(x) \in [0,1]$ is called the grade of membership of x in (X, μ_X). For a fuzzy set (X, μ_X) and $\alpha \in [0,1]$, $X_\alpha := \{x \in X : \mu_X(x) \geqslant \alpha\}$ is called the (closed) α- *cut*. A *fuzzy map* from (X, μ_X) to (Y, μ_Y) written as $f : (X, \mu_X) \to (Y, \mu_Y)$ is an ordinary map $f : X \to Y$ such that $\mu_Y f \geqslant \mu_X$, where $\mu_Y(f(x)) \geqslant \mu_X(x)$, for every $x \in X$. In this case we call the following triangle is a fuzzy triangle.

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[d, "\mu_X"'] \arrow[dr, phantom, "\leq"] & Y \arrow[dl, "\mu_Y"] \\ {[0,1]} & \end{tikzcd}$$

Fuzzy sets together with fuzzy maps between them form a category denoted by **FSet**.

2.6. Definition. The group Perm($\mathbb{D}$) together with a map $\nu : \mathrm{Perm}(\mathbb{D}) \to [0,1]$ is called a *fuzzy group* if

(i) its multiplication is a fuzzy map, this is, for every $\pi, \pi' \in \mathrm{Perm}(\mathbb{D})$, $\nu(\pi\pi') \geqslant \nu(\pi) \wedge \nu(\pi')$, that is the following diagram is a fuzzy triangle.

$$\begin{tikzcd} \mathrm{Perm}(\mathbb{D}) \times \mathrm{Perm}(\mathbb{D}) \arrow[r, "\lambda_\pi"] \arrow[d, "\nu \wedge \nu"'] & \mathrm{Perm}(\mathbb{D}) \arrow[dl, "\nu"] \\ {[0,1]} & \end{tikzcd}$$

(ii) $\nu(id) = 1$;
(iii) $\nu(\pi) = \nu(\pi^{-1})$.

3. Category of fuzzy nominal sets

In this section, we are going to fuzzify the notion of nominal sets according to the universal algebraic approach.

3.1. Theorem. *Suppose $(\mathbb{D}, \mu_{\mathbb{D}})$ is a fuzzy set. The pair $(\mathrm{Perm}(\mathbb{D}), \nu_{\mathrm{supp}})$ is a fuzzy group, in which $\nu_{\mathrm{supp}} : \mathrm{Perm}(\mathbb{D}) \to [0,1]$ mapping each $\pi \in \mathrm{Perm}(\mathbb{D})$ to $\bigwedge_{d \in \mathrm{supp}\,\pi} \mu_{\mathbb{D}}(d)$.*

Note. In this section, we take $(\mathrm{Perm}(\mathbb{D}), \nu_{\mathrm{supp}})$ to be a fuzzy group.

3.2. Definition. The fuzzy set (X, μ_X) is a fuzzy nominal set if X is a nominal set and the family $\{\lambda_\pi\}_{\pi \in \mathrm{Perm}(\mathbb{D})}$ of unary operations are fuzzy maps. Meaning that the following diagram is a fuzzy triangle, for every $\pi \in \mathrm{Perm}(\mathbb{D})$.

$$\begin{tikzcd} X \arrow[r, "\lambda_\pi"] \arrow[d, "\mu_X"'] \arrow[dr, phantom, "\leq"] & X \arrow[dl, "\mu_X"] \\ {[0,1]} & \end{tikzcd}$$

More explicitly, the fuzzy set (X, μ_X) is a fuzzy nominal set if X is a nominal set and $\mu_X(\pi x) \geqslant \mu_X(x)$, for every $\pi \in \mathrm{Perm}(\mathbb{D})$ and $x \in X$.
Fuzzy nominal sets with fuzzy equivariant maps between them form a category, denoted by, **F-Nom**.

3.3. Example. (i) Suppose (X, μ_X) is a fuzzy set and X is a discrete nominal set, so (X, μ_X) is a fuzzy nominal set.
(ii) The pair $(\mathrm{Perm}(\mathbb{D}), \mu_{\mathrm{Perm}(\mathbb{D})})$ where $\mu_{\mathrm{Perm}(\mathbb{D})}(\pi) = \bigvee_{d \in \mathrm{supp}\,\pi} \mu_{\mathbb{D}}(d)$ is a fuzzy nominal set. Notice that, by Example 2.4(ii), $\mathrm{Perm}(\mathbb{D})$ is a nominal set. Also, since $\mathrm{supp}\,\pi\pi_1 = \mathrm{supp}\,\pi \cup \mathrm{supp}\,\pi_1$, we get that $\mu_{\mathrm{Perm}(\mathbb{D})}(\pi\pi_1) \geqslant \mu_{\mathrm{Perm}(\mathbb{D})}(\pi_1)$, for every $\pi, \pi_1 \in \mathrm{Perm}(\mathbb{D})$.

3.4. Lemma. *Let (X, μ_X) be a fuzzy nominal set. Then, for every $\pi \in \mathrm{Perm}(\mathbb{D})$ and every $x \in X$, we have:*
(i) $\mu_X(\pi x) = \mu_X(x)$.
(ii) $\mu_X(\pi x) = \mu_X(\pi^{-1} x)$.

From Lemma 3.4, we obtain the following corollary.

3.5. Corollary. (i) *The fuzzy set (X, μ_X) is a fuzzy nominal set if and only if $\mu_X(\pi x) = \mu_X(x)$, for every $x \in X$ and $\pi \in \mathrm{Perm}(\mathbb{D})$.*
(ii) *For an indecomposable (cyclic, single orbit) nominal set X, the fuzzy set (X, μ_X) is a fuzzy nominal set if and only if μ_X is constant.*
(iii) *Consider $[0,1]$ to be a discrete nominal set, and suppose (X, μ_X) is a fuzzy nominal set. Then the map $\mu_X : X \to [0,1]$ is equivariant.*

3.6. Lemma. *Suppose the pair (X, μ_X) is a fuzzy nominal set, then X_α is an equivariant subset of X, for every $\alpha \in [0,1]$.*

3.7. Lemma. *The pair* (X, μ_X) *is a fuzzy nominal set if and only if* $(X/\sim, \mu_{X/\sim})$ *is a fuzzy nominal set where* $\mu_{X/\sim}(\mathrm{orb}x) = \bigvee_{x' \in \mathrm{orb}x} \mu_X(x')$.

3.8. Remark. Let $(\mathbb{D}, \mu_{\mathbb{D}})$ be a fuzzy nominal set. Then,
(i) the pair $(X, \mu_X^{\mathrm{supp}})$ is a fuzzy nominal set, where $\mu_X^{\mathrm{supp}}(x) = \bigwedge_{d \in \mathrm{supp}\, x} \mu_{\mathbb{D}}(d)$.
(ii) if $f : X \longrightarrow Y$ is equivariant, then f is a fuzzy equivariant map from $(X, \mu_X^{\mathrm{supp}})$ to $(Y, \mu_Y^{\mathrm{supp}})$.
(iii) the pair (X, μ_X), where $\mu_X(x) = \bigvee_{d \in \mathrm{supp}\, x} \mu_{\mathbb{D}}(d)$, is a fuzzy nominal set.

3.9. Proposition. *Suppose* (X, μ_X) *is fuzzy set and* X *is a nominal set* $x \in X$*. Then,* (X, μ_X) *is a fuzzy nominal set if and only if* X *is a nominal set and* $\mu_X(\pi x) \geqslant \mu_X(x)$*, for every* $\pi \in \mathrm{Perm}(\mathbb{D})$ *with* $\mathrm{supp}\, x \cap \mathrm{supp}\, \pi \neq \emptyset$.

From Corrollary 3.5(i) and Proposition 3.9 we obtain the following result.

3.10. Corollary. *Suppose* $(\mathbb{D}, \mu_{\mathbb{D}})$ *is a fuzzy set and* $\mathbb{D}$ *is a nominal set and* $d \in \mathbb{D}$*. Then* $(\mathbb{D}, \mu_{\mathbb{D}})$ *is a fuzzy nominal set if and only if* $\mu_{\mathbb{D}}(\pi d) \geqslant \mu_{\mathbb{D}}(d)$*, for every* $\pi \in \mathrm{Perm}(\mathbb{D})$ *and* $d \in \mathrm{supp}\, \pi$.

3.11. Lemma. $(\mathbb{D}, \mu_{\mathbb{D}})$ *is a fuzzy nominal set if and only if* $(\mathcal{P}_{\mathrm{f}}(\mathbb{D}), \mu_{\mathcal{P}_{\mathrm{f}}(\mathbb{D})})$ *is a fuzzy nominal set, where* $\mu_{\mathcal{P}_{\mathrm{f}}(\mathbb{D})}(A) = \bigwedge_{d \in A} \mu_{\mathbb{D}}(d)$*, for every* $A \in \mathcal{P}_{\mathrm{f}}(\mathbb{D})$.

4. Related Categories

In this section we consider the relationships among the categories **F-Nom**, **Nom** and **FSet**. In particular, we show there are a number of adjunctions between these categories.

4.1. Theorem. *The assignment* $G : \mathbf{Nom} \to \mathbf{F\text{-}Nom}$ *assigning each object* X *to* $(X, G\mu_X)$*, in which* $G\mu_X(x) = \mu_X^{\mathrm{supp}}(x)$*, and each fuzzy equivarinat map* f *to* $G(f)(x) = f(x)$ *is a functor.*

4.2. Remark. (i) The assignment $K : \mathbf{Nom} \to \mathbf{F\text{-}Nom}$, assigning each nominal set X to $K(X) = (X, 1)$ and each equivariant map $f : X \to Y$ to $K(f) = f$, is a functor and it is a right adjoint to the forgetful functor $U : \mathbf{F\text{-}Nom} \to \mathbf{Nom}$.
(ii) The assignment $H : \mathbf{Nom} \to \mathbf{F\text{-}Nom}$ defined by by $H(X) = (X, 0)$, for every nominal set X, and $H(f) = f$, for every equivariant map $f : X \to Y$, is a functor and it is a left adjoint to the forgetful functor.

4.3. Corollary. *We use Remark 4.2 and we have that the limits and colimits in* **F-Nom** *are obtained at the level of* **Nom**.

4.4. Remark. (i) The assignment $\Delta : \mathbf{FSet} \to \mathbf{F\text{-}Nom}$, assigning each fuzzy set (X, μ_X) to $\Delta(X, \mu_X) = (X, 1)$ where X is a discrete nominal set and each fuzzy map $f : (X, \mu_X) \to (Y, \mu_Y)$ to fuzzy equivariant map $\Delta(f) = f$, is a functor.
(ii) The assignment $-/\mathrm{Perm}(\mathbb{D}) : \mathbf{F\text{-}Nom} \to \mathbf{FSet}$, assigning each fuzzy nominal set (X, μ_X) to $(X, \mu_X)/\mathrm{Perm}(\mathbb{D}) = (X/\mathrm{Perm}(\mathbb{D}), 0)$ and each fuzzy equivariant map $f : (X, \mu_X) \to (Y, \mu_Y)$ to fuzzy map $f/\mathrm{Perm}(\mathbb{D})(\mathrm{orb}x) = \mathrm{orb}f(x)$, is a functor.

4.5. Corollary. *The functor* Δ *is the right adjoint to the functor* $-/\mathrm{Perm}(\mathbb{D})$.

5. Fuzzy freshness relation

In this section we define fuzzy freshness relation and investigate its properties.

5.1. Definition. [2] A function $\sigma : X \times X \to [0, 1]$ is called *fuzzy relation* on a non-empty set X. A fuzzy relation σ on a non-empty set X is said to be a *fuzzy equivalence relation* on X if it satisfies the following for every $x, y, z \in X$:
(i) Fuzzy reflexive: $\sigma(x, x) = 1$,
(ii) Fuzzy symmetric: $\sigma(x, y) = \sigma(y, x)$,
(iii) Fuzzy transitive: $\sigma(x, y) \geqslant \bigvee_{z \in X} (\sigma(x, z) \wedge \sigma(z, y))$.

5.2. Definition. [9] Given nominal sets X and Y and elements $x \in X$ and $y \in Y$, we write $x \sharp y$ and say that *x is fresh for y* if x and y have disjoint supports that is

$$x \sharp y \iff \operatorname{supp} x \cap \operatorname{supp} y = \emptyset.$$

5.3. Definition. Suppose X is a nominal set and $(\mathbb{D}, \mu_{\mathbb{D}})$ is a fuzzy nominal set. The function $\mu^{\mathrm{supp}}_{X \times X} : X \times X \to [0,1]$ defined by $\mu^{\mathrm{supp}}_{X \times X}(x,y) = \bigwedge_{d \in \operatorname{supp} x \cap \operatorname{supp} y} \mu_{\mathbb{D}}(d)$ is called a *fuzzy freshness relation* on nominal set X.

5.4. Remark. Suppose $(\mathbb{D}, \mu_{\mathbb{D}})$ is a fuzzy nominal set and X is a nominal set. Then
(i) $(X \times X, \mu^{\mathrm{supp}}_{X \times X})$ is a fuzzy nominal set.
(ii) $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy symmetric relation.
(iii) if $x \sharp y$, then $\mu^{\mathrm{supp}}_{X \times X}(x,y) = 1$.

5.5. Lemma. *Suppose X is a nominal set and $(\mathbb{D}, \mu_{\mathbb{D}})$ is a fuzzy nominal set. Then $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy reflexive relation if and only if $\mu^{\mathrm{supp}}_{X \times X}(x,y) = 1$, for every $x, y \in X$.*

5.6. Corollary. *The fuzzy relation $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy equivalence relation if and only if $\mu^{\mathrm{supp}}_{X \times X}(x,y) = 1$, for every $x, y \in X$.*

5.7. Lemma. *The fuzzy relation $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy reflexive relation if and only if $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy equivalence relation.*

5.8. Lemma. *Suppose X is a nominal set and $(\mathbb{D}, \mu_{\mathbb{D}})$ is a fuzzy nominal set with $\mu_{\mathbb{D}} : \mathbb{D} \to [0,1)$. Then $x \sharp y$ if and only if $\mu^{\mathrm{supp}}_{X \times X}(x,y) = 1$.*

5.9. Corollary. *Suppose X is a nominal set and $(\mathbb{D}, \mu_{\mathbb{D}})$ is a fuzzy nominal set with $\mu_{\mathbb{D}} : \mathbb{D} \to [0,1)$. Then $x \sharp y$ if and only if $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy equivalence relation.*

5.10. Lemma. *Suppose X is a nominal set and $(\mathbb{D}, \mu_{\mathbb{D}})$ is a fuzzy nominal set with $\mu_{\mathbb{D}} : \mathbb{D} \to [0,1)$. If $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy reflexive(equivalence) relation, then X is a discrete nominal set.*

5.11. Corollary. *Suppose X is a nominal set and $(\mathbb{D}, \mu_{\mathbb{D}})$ is a fuzzy nominal set with $\mu_{\mathbb{D}} : \mathbb{D} \to [0,1)$. Then $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy reflexive(equivalence) relation on X if and only if X is a discrete nominal set.*

5.12. Lemma. *Suppose (X, μ_X) and $(\mathbb{D}, \mu_{\mathbb{D}})$ are fuzzy nominal sets. If $\mu^{\mathrm{supp}}_{X \times X}$ is a fuzzy reflexive(equivalence) relation, then the equivariant map* supp_X *is a fuzzy equivariant map from (X, μ_X) to $(\mathcal{P}_{\mathrm{f}}(\mathbb{D}), \mu_{\mathcal{P}_{\mathrm{f}}(\mathbb{D})})$.*

References

[1] Adámek. J and Herrlich. H and Strecker. G. E, Abstract and concrete categories. The joy of cats, (2004).
[2] Pal, Pavel, Sujit Kumar Sardar, and Rajlaxmi Mukherjee. "Fuzzy Operands of Semigroups." Fuzzy Information and Engineering 6, no. 2 (2014): 235-244.
[3] Ebrahimi, M.M. and Mahmoudi, M., The category of M-sets, Ital. J. Pure Appl. Math. 9 (2001), 123-132.
[4] Fraenkel. J. A, Der begriff definit und die unabhangigkeit des auswahlsaxioms, Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-mathematische Klasse, (1922) pages 253257.
[5] Gabbay, M. and Pitts, A., A new approach to abstract syntax with variable binding, Form. Asp. Comput. (13)(3-5)(2002), 341-363.
[6] Haddadi, M. "Fuzzy Acts over Fuzzy Semigroups and Sheaves." Iranian Journal of Fuzzy Systems 11, no. 4 (2014): 61-73.
[7] Boixader, D. and Recasens, J., "Fuzzy actions," Fuzzy Sets Syst.", 339 (2018), 17-30
[8] Kilp, M., Knauer, U. and Mikhalev, A., " Monoids, Acts and Categories," Walter de Gruyter, Berlin, New York, 2000.
[9] Pitts, A., "Nominal sets, Names and Symmetry in Computer Science," Cambridge University Press, 2013.
[10] Pitts, Andrew. "Nominal presentation of cubical sets models of type theory." (2015).

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