NEW GENERALIZATION OF DARBO'S FIXED POINT THEOREM VIA α-ADMISSIBLE SIMULATION FUNCTIONS WITH APPLICATION

HOSSEIN MONFARED AND ALI FARAJZADEH

Abstract. In this paper, firstly, we introduce α_μ-admissible Z_μ-contraction and α_μ-admissibleN_μ-contraction via stimulation functions. Secondly, we prove some new fixed point theorems for defined class of contractions via α-admissible stimulation mappings. Our results extend some existing results. Moreover, some examples and an application to functional integral equations are given to support the obtained results.

keywords: Measure of non-compactness, simulation functions, α-admissible mappings, Fixed point.

1. Introduction and Preliminaries

Schauder fixed point theorem is one of the useful and important tools in analysis. In 1955, Darbo [2], by using the concept of a measure of non-compactness, proved the fixed point property for known contraction on a closed, bounded and convex subset of Banach spaces. Darbo's fixed point plays a key role in nonlinear analysis especially in proving the existence of solutions for a lot of classes of nonlinear equations. Since then, some generalizations of Darbo's fixed point theorem have been proved. For example, we refer the reader to [3–8] and the references therein. Recently, Jianhua Chen *et al.* [1] proved some new generalizations of Darbo's fixed point theorem by using the notion of simulation function that Khojasteh *et al.* [13] proposed it.

In this paper, we investigate the existence of fixed points of certain mappings via α_μ-admissible simulation functions for α-set contraction on a closed, bounded and convex subset of Banach spaces.

Throughout this paper, by $\mathbb{N}$, $\mathbb{R}_+$ and $\mathbb{R}$, respectively, denote the set of all positive integers, non-negative real numbers and real numbers. Now, let us recall some basic concepts, notations and known results which will be used in the sequel. We let E be a Banach space with the norm $\|.\|$ and ϑ be the zero element in E. The closed ball centered at x with radius r is denoted by $B(x, r)$, by simply B_r if $x = 0$. If X is a nonempty subset of E, then we denote by X and $\bar{\text{co}}(X)$ the closure and closed convex hull of X, respectively. Moreover, let M_E be the family of all nonempty bounded subsets of E and by N_E the subfamily consisting of all relatively compact subsets of E. In [11], Bana's *et al.* gave the concepts of a measure of non-compactness.

Definition 1.1. *A mapping $\mu : M_E \to R_+$ is said to be a measure of non-compactness in E if it satisfies the following conditions:*

(1) The family $\ker\mu = \{\mathrm{X} \in \mathrm{M_E} : \mu(\mathrm{X}) = 0\}$ is nonempty and $\ker\mu \subseteq \mathrm{N_E}$;
(2) $X \subseteq Y \Rightarrow \mu(X) \le \mu(Y)$;
(3) $\mu(\bar{\text{co}}X) = \mu(\bar{X}) = \mu(X)$;
(4) $\mu(\lambda X + (1-\lambda)Y) \le \lambda\mu(X) + (1-\lambda)\mu(Y)$ for all $\lambda \in [0, 1]$.
(5) If $\{X_n\}$ is a sequence of closed sets from M_E such that $X_{n+1} \subseteq X_n$ for $n = 1, 2, ...$ and $\lim_{n\to\infty} \mu(X_n) = 0$, then the intersection set $X_\infty = \cap_{n=1}^{\infty} X_n$ is nonempty.

The family ker μ described in (1) in Definition 1.1 is said to be the kernel of the measure of non-compactness μ. Observe that the intersection set X_∞ from (4) is a member of the family ker μ. In fact, since $\mu(X_\infty) \le \mu(X_n)$ for any n, we infer that $\mu(X_\infty) = 0$. This yields that $X_\infty \in \ker\mu$.

Theorem 1.1. (Schauder fixed point principle) Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E. Then each continuous and compact map $T : \Omega \to \Omega$ has at least one fixed point in the set Ω.

Obviously the above formulated theorem constitutes the well known Schauder fixed point principle. It's generalization, called the Darbo's fixed point theorem, is formulated below.

Theorem 1.2. ([1, Darbo's fixed point theorem]) Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous mapping. Assume that there exists a constant $k \in [0, 1)$ such that

$$\mu(TX) \le k\mu(X).$$

for any nonempty subset X of Ω, where μ is a measure of non-compactness defined in E. Then T has a fixed point in the set Ω.

In order to prove our fixed point theorems, we need some the following related concepts. First of all, we recall the definition of the class of function as follows.

Definition 1.2. ([12, Khan *et al.*]) An altering distance function is a continuous, non-decreasing mapping $\varphi : [0,\infty) \to [0,\infty)$ such that $\varphi^{-1}(\{0\}) = \{0\}$.

However, in [13], the authors slightly modified the definition of simulation function which introduced by Khojasteh et al. [14] and enlarged the family of all simulation functions.

Definition 1.3. ([13]) *A function* $\sigma : [0,\infty) \times [0,\infty) \to \mathbb{R}$ *is said to be simulation if it fulfils:*
(σ_1) $\sigma(0,0) = 0$*;*
(σ_2) $\sigma(t,u) < u - t$ for all $t, u > 0$*;*
(σ_3) *if* $\{t_n\}$, $\{u_n\}$ *are sequences in* $(0,\infty)$ *such that* $\lim_{n\to\infty} t_n = \lim_{n\to\infty} u_n > 0$, *then*

$$\limsup_{n\to\infty} \sigma(t_n, u_n) < 0. \tag{1}$$

Let Z be the collection of all simulation functions $\sigma : [0,\infty) \times [0,\infty) \to \mathbb{R}$. On account of the property (σ_2), we conclude that

$$\sigma(t,t) < 0 \text{ for all } t > 0. \tag{2}$$

Thereupon, the authors [14] give the following example to illustrate that every simulation function in the original Khojasteh *et al.*'s sense (Definition 1.2) is also a simulation function in Roldn-Lpez- de-Hierro et al.s sense (Definition 1.3), but the converse is not true.

Example 1.1. *Let* $\sigma : [0,\infty) \times [0,\infty) \to \mathbb{R}$ *be a mapping such that* $\sigma(t,u) = \frac{u}{2} - t$ *for all* $t, u \in [0,\infty)$. *It is obvious that* σ *is a simulation function. For more examples of simulation functions in [13, 21].*

Definition 1.4. ([1]) A function $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$ is said to be generalized simulation if:
$\zeta(t,s) \leq s - t$ for all $t, s > 0$;

Let N be the family of all generalized simulation functions $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$.

Definition 1.5. *Let* $f : X \to X$ *and* $\alpha : X \times X \to (-\infty, +\infty)$. *We say that* f *is an* α*-admissible mapping if* $\alpha(x,y) \geq 1$ *implies* $\alpha(fx, fy) \geq 1$, *for all* $x, y \in X$, In what follows we recall the notion of (triangular) α-orbital admissible, introduced by Popescu [20], that is inspired from [19].

Definition 1.6. ([20]) For a fixed mapping $\alpha : M \times M \to [0, \infty)$, we say that a self-mapping $T : M \to M$ is an α-orbital admissible if

$$(O1) \;\; \alpha(u, Tu) \geq 1 \Rightarrow \alpha(Tu, T^2u) \geq 1.$$

Let $\mathcal{A}$ be the collection of all α-orbital admissible $T : M \to M$.

In addition, T is called triangular α-orbital admissible if T is α-orbital admissible and

$$(O2) \;\; \alpha(u, v) \geq 1 \text{ and } \alpha(v, Tv) \geq 1 \Rightarrow \alpha(u, Tv) \geq 1$$

Let $\mathcal{O}$ be the collection of all triangular α-orbital admissible $T : M \to M$.

Definition 1.7. ([1]) Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. We say that T is a Z_μ-contraction if there exists $\xi \in Z$ such that

$$\xi(\mu T(X)), \mu(X) \geq 0. \tag{3}$$

for any nonempty subset X of Ω, where μ is an arbitrary measure of non-compactness.

Now, we observe some useful properties of Z_μ-contractions in Banach spaces.

Remark 1.1. ([1]) If T is a Z_μ-contraction with respect to $\xi \in Z$, then

$$\mu(T(X)) < \mu(X) \tag{4}$$

for any nonempty subset X of Ω. To prove it, applying ($\xi(2)$) and (3), we have

$$0 \leq \xi(\mu(T(X)), \mu(X)) < \mu(X) - \mu(T(X)).$$

Hence, (4) holds. Next, we prove the following fixed point theorem.

Theorem 1.3. ([1]) Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. If T is a Z_μ-contraction with respect to $\xi \in Z$. Then T has at least one fixed point in Ω .

Definition 1.8. ([1]) Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. We say that T is a

N_μ-contraction if there exists $\zeta \in N$ such that

$$\xi(\mu T(X)), \kappa\mu(X)) \geq 0. \tag{5}$$

for any nonempty subset X of Ω, where μ is an arbitrary measure of non-compactness, and $\kappa : [0, \infty) \to \mathbb{R}_+$ is nondecreasing on $\mathbb{R}_+$ and such that $\lim_{n\to\infty} \kappa^n(t) = 0$ for each $t > 0$.

Now, we get some useful properties of N_μ-contractions in Banach spaces.

Remark 1.2. (1) By the definition of generalized stimulation functions, it is clear that a generalized stimulation function must verify $\zeta(r, r) \leq 0$ for all $r > 0$.
(2) If T is N_μ-contraction with respect to $\zeta \in N$, then

$$\mu(T(X)) \leq \kappa(\mu(X)) \tag{6}$$

for any nonempty subset X of Ω. To prove it, applying Definition 1.8, we have

$$0 \leq \xi(\mu(TX), \kappa(\mu(X))) \leq \kappa(\mu(X)) - \mu(TX).$$

Hence, (6) holds.

2. Main result and Fixed point theorems via α-admissible stimulation functions

In order to prove our fixed point theorems, we need some the following related concepts. First of all, we recall the definition of the class of function as follows.

Definition 2.1. *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator, and $\alpha : \mu(M_E) \times \mu(M_E) \to (-\infty, +\infty)$. We say that T is an α_μ-admissible mapping if*

$$\alpha(\mu(X), \mu(Y)) \geq 1 \Rightarrow \alpha(\mu(TX), \mu(TY)) \geq 1,$$

for any nonempty subsets X and Y of Ω, where μ is an arbitrary measure of non-compactness.

Definition 2.2. *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous and α_μ-admissible operator. We say that T is an α_μ-admissible Z_μ-contraction if there exists $\xi \in Z$ such that*

$$\xi(\alpha(\mu(X), \mu(TX))\mu T(X)), \mu(X)) \geq 0. \tag{7}$$

Remark 2.1. *If* $\alpha(x, y) = 1$*, then* T *turns into a* Z_μ*-contraction with respect to* ξ*.*

Remark 2.2. *If* T *is an* α_μ*-admissible* Z_μ*-contraction with respect to* ξ*, then*

$$\alpha(\mu(X), \mu(TX))\mu(TX) < \mu(X) \text{ for all } X \subseteq \Omega \text{ such that } \mu(X) > 0. \tag{8}$$

To prove the assertion, we assume that $X \subseteq \Omega$*. If* $\mu(TX) = 0$*, then*

$$\alpha(\mu(X), \mu(TX))\mu(TX) = 0 < \mu(X).$$

Otherwise, $\mu(TX) > 0$*. If* $\alpha(\mu(X), \mu(TX)) = 0$*, then the inequality is satisfied trivially. So assume that* $\alpha(\mu(X), \mu(TX)) > 0$ *and applying* $\xi(2)$ *with* (7)*, we derive that*

$$0 \leq \xi(\alpha(\mu(X), \mu(TX))\mu(TX), \mu(X)) < \mu(X)) - \alpha(\mu(X), \mu(TX))\mu(TX).$$

so (8) *holds.*

Theorem 2.1. *Let* Ω *be a nonempty, bounded, closed and convex subset of a Banach space* E *and let* $T : \Omega \to \Omega$ *be a continuous operator. If* T *is an* α_μ*-admissible* Z_μ*-contraction with respect to* $\xi \in Z$*, and there exits* $X_0 \subseteq \Omega$ *such that* X_0 *be a closed and convex,* $TX_0 \subseteq X_0$ *and* $\alpha(\mu(X_0), \mu(TX_0)) \geq 1$ *then* T *has at least one fixed point in* Ω *.*

Proof. Let $X_0 \subseteq \Omega$ be such that $\alpha(\mu(X_0), \mu(TX_0)) \geq 1$, and $TX_0 \subseteq X_0$, and define a sequence $\{X_n\}$ as follows:
$X_n = \bar{\text{co}}(TX_{n-1})$, for all $n \geq 1$.
So by induction we get

$$X_n \subseteq X_{n-1} \text{ and } TX_n \subseteq X_n.$$

since by hypothesis we have

$$TX_0 \subseteq X_0,$$

so we have

$$X_1 = \bar{\text{co}}(TX_0) \subseteq \bar{\text{co}}(X_0) = X_0.$$

Now suppose that $X_{n+1} \subseteq X_n$, therefore we get

$$X_{n+2} = \bar{\text{co}}(TX_{n+1}) \subseteq \bar{\text{co}}(TX_n) = X_{n+1},$$

and

$$TX_{n+1} \subseteq TX_n \subseteq \bar{\text{co}}(TX_n) = X_{n+1}.$$

If there exists natural number n_0 such that $\mu(X_{n_0}) = 0$, then X_{n_0} is compact and since $TX_{n_0} \subset X_{n_0}$. Thus Theorem 1.1 implies that T has a fixed point. Next, we suppose that $\mu(X_n) > 0$ for all $n \geq 0$.
Regarding that T is α_μ-admissible, we derive

$$\begin{aligned}\alpha(\mu(X_0), \mu(X_1)) &= \alpha(\mu(X_0), \mu(\bar{\text{co}}(TX_0))) = \alpha(\mu(X_0), \mu(TX_0)) \geq 1 \\ &\Rightarrow \alpha(\mu(TX_0), \mu(TX_1)) = \alpha(\mu(X_1), \mu(X_2)) \geq 1.\end{aligned}$$

Recursively, we obtain that

$$\alpha(\mu(X_n), \mu(X_{n+1})) \geq 1, \text{ for all } n \geq 0. \tag{9}$$

On the other hand by our assumptions and (3), we get

$$\begin{aligned}\xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \mu(X_n)) &= \xi(\alpha(\mu(X_n), \mu(\bar{\text{co}}(TX_n))\mu(\bar{\text{co}}(TX_n)), \mu(X_n)) \\ &= \xi(\alpha(\mu(X_n), \mu(TX_n))\mu(TX_n), \mu(X_n)) \geq 0.\end{aligned} \tag{10}$$

Based on Remark 2.2, we can get

$$\xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \mu(X_n)) < \mu(X_n) - \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}). \tag{11}$$

From (9), (10) and (11), we infer that

$$\mu(X_{n+1}) \leq \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}) < \mu(X_n). \tag{12}$$

Hence, $\{\mu(X_n)\}$ is a decreasing sequence of positive real numbers. Thus, there exists $r \geq 0$, such that $\mu(X_n) \to r$ as $n \to \infty$. Next, we show that $r = 0$. Suppose, to the contrary, that $r > 0$. Also by (12) we have

$$\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}) \to r > 0 \text{ as } n \to \infty.$$

Applying the axiom $\sigma(3)$ in Definition 1.3 to the sequences:

$$\{t_n = \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1})\} \text{ and } \{s_n = \mu(X_n)\}$$

(which have the same limit $r > 0$ and verify $t_n < s_n$ for all n), it follow that

$$\limsup_{n\to\infty} \xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \mu(X_n)) = \limsup_{n\to\infty} \xi(t_n, s_n) < 0.$$

which contradicts (10). We get $r = 0$, and hence $\mu(X_n) \to 0$ as $n \to \infty$. Now since $\{X_n\}$ is a nested sequence, in view of (5) of Definition 1.1, we conclude that

$X_\infty = \cap_{n=1}^{\infty} X_n$ is a nonempty, closed and convex subset of Ω. Moreover, we know that X_∞ belongs to ker μ. So X_n is compact and invariant under the mapping T. Consequently, Theorem 1.1 implies that T has a fixed point in X_∞ . Since $X_\infty \subseteq \Omega$, then the proof is completed. □

Corollary 2.1. (Theorem 2.1 in [1]) Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. If T is a Z_μ-contraction with respect to $\xi \in Z$, then T has at least one fixed point in Ω .

Proof. In Theorem **??** let $\alpha(x, y) = 1$. □

3. Fixed point theorems via α-admissible generalized stimulation functions

Definition 3.1. *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous and α_μ-admissible operator. We say that T is an α_μ-admissible N_μ-contraction if there exists $\xi \in N$ such that*

$$\xi(\alpha(\mu(X), \mu(TX))\mu T(X)), \kappa\mu(X)) \geq 0. \tag{13}$$

for any nonempty subset X of Ω, where μ is an arbitrary measure of non-compactness and $\kappa : [0, \infty) \to \mathbb{R}_+$ is nondecreasing on $\mathbb{R}_+$ and such that $\lim_{n\to\infty} \kappa^n(t) = 0$, for each $t > 0$.

Remark 3.1. *If $\alpha(x, y) = 1$, then T turns into a N_μ-contraction with respect to ξ.*

Remark 3.2. *If T is an α_μ-admissible N_μ-contraction with respect to ξ, then*

$$\alpha(\mu(X), \mu(TX))\mu(TX) \leq \kappa(\mu(X)) \text{ for all } X \subseteq \Omega \text{ such that } \mu(X) > 0. \tag{14}$$

To prove the assertion, we assume that $X \subseteq \Omega$. If $\mu(TX) = 0$, then

$$\alpha(\mu(X), \mu(TX))\mu(TX) = 0 \leq \kappa(\mu(X)).$$

Otherwise, $\mu(TX) > 0$. If $\alpha(\mu(X), \mu(TX)) = 0$, then the inequality is satisfied trivially. So assume that $\alpha(\mu(X), \mu(TX)) > 0$ and applying (13), we derive that

$$0 \leq \xi(\alpha(\mu(X), \mu(TX))\mu(TX), \kappa(\mu(X))) \leq \kappa(\mu(X)) - \alpha(\mu(X), \mu(TX))\mu(TX).$$

so (14) holds.

Next, we prove the following fixed point theorem.

Theorem 3.1. *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. If T is an α_μ-admissible N_μ-contraction with respect to $\xi \in Z$, and there exits $X_0 \subseteq \Omega$ such that X_0 be a closed and convex, $TX_0 \subseteq X_0$ and $\alpha(\mu(X_0), \mu(TX_0)) \geq 1$ then T has at least one fixed point in Ω .*

Proof. Let $X_0 \subseteq \Omega$ be such that $\alpha(\mu(X_0), \mu(TX_0)) \geq 1$, and $TX_0 \subseteq X_0$, and define a sequence $\{X_n\}$ as follows:
$X_n = \bar{\text{co}}(TX_{n-1})$, for all $n \geq 1$.
If there exists natural number n_0 such that $\mu(X_{n_0}) = 0$, then X_{n_0} is compact and since $TX_{n_0} \subset X_{n_0}$. Thus Theorem 1.1 implies that T has a fixed point. Next, we suppose that $\mu(X_n) > 0$ for all $n \geq 0$.
Regarding that T is α_μ-admissible, we derive

$$\begin{aligned} \alpha(\mu(X_0), \mu(X_1)) &= \alpha(\mu(X_0), \mu(\bar{\text{co}}(TX_0))) = \alpha(\mu(X_0), \mu(TX_0)) \geq 1 \\ &\Rightarrow \alpha(\mu(TX_0), \mu(TX_1)) = \alpha(\mu(X_1), \mu(X_2)) \geq 1. \end{aligned}$$

Recursively, we obtain that

$$\alpha(\mu(X_n), \mu(X_{n+1})) \geq 1, \text{ for all } n = 0, 1, \ldots \tag{15}$$

On the other hand by our assumptions and (5), we get

$$\begin{aligned} \xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \kappa(\mu(X_n))) &= \xi(\alpha(\mu(X_n), \mu(\bar{\text{co}}(TX_n))\mu(\bar{\text{co}}(TX_n)), \kappa(\mu(X_n))) \\ &= \xi(\alpha(\mu(X_n), \mu(TX_n))\mu(TX_n), \kappa(\mu(X_n))) \geq 0. \end{aligned} \tag{16}$$

Based on Remark 3.2, we can get

$$\xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \kappa(\mu(X_n))) < \kappa(\mu(X_n)) - \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}). \tag{17}$$

From (15), (16) and (17), we infer that

$$\mu(X_{n+1}) \leq \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}) < \kappa(\mu(X_n)) \text{ for all } n \in \mathbb{N}. \tag{18}$$

Since $\kappa : [0, \infty) \to \mathbb{R}_+$ is nondecreasing, we can get

$$\mu(X_{n+1}) \leq \kappa(\mu(X_n)) \leq \kappa(\kappa(\mu(X_{n-1}))) \leq \cdots \leq \kappa^n(\mu(X_0)) \tag{19}$$

In (19), Letting $n \to \infty$, we have

$$\lim_{n\to\infty} \mu(X_{n+1}) = 0.$$

Now since $\{X_n\}$ is a nested sequence, in view of (5) of Definition 1.1, we conclude that $X_\infty = \cap_{n=1}^{\infty} X_n$ is a nonempty, closed and convex subset of Ω. Moreover, we know that X_∞ belongs to $\ker \mu$. So X_n is compact and invariant under the mapping T. Consequently, Theorem 1.1 implies that T has a fixed point in X_∞ . Since $X_\infty \subseteq \Omega$, then the proof is completed.

□

Corollary 3.1. (Theorem 3.1 in [1]) Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. If T is a N_μ-contraction with respect to $\xi \in Z$, then T has at least one fixed point in Ω .

Proof. In Theorem **??** let $\alpha(x, y) = 1$. □

Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors have read and approved the final manuscript.

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(H. Monfared) Department of Mathematics, Germi Branch, Islamic Azad University, Germi, Iran

E-mail address: monfared.h@gmail.com

(Ali Farajzadeh) Department of Mathematics, Kermanshah University, Kermanshah, Iran

E-mail address: farajzadehali@gmail.com