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A BI-OBJECTIVE MODEL TO ESTIMATE THE DEMANDS OF NODES IN THE P-MEDIAN PROBLEM

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Abstract. In this paper, applying a least squared approach, a new inverse bi-objective model to the p-median problem is proposed to estimate the demands of nodes where the locations of p facilities with the number of visited customers by each one and the target values for nodes' demands are previously available. The purpose is to determine the allocation solution in such away that the sum of squared errors between the target and the estimated values as well as the transportation cost would be minimized. The resulted bi-objective problem is solved by using the ϵ-constraint algorithm and a numerical example is provided to verify the added value of the proposed model.

1. Introduction

Locating p facilities and allocating the clients' demands with the purpose of minimizing the total transportation cost, named as the p-median, has been always one of the main network issues. Such exact methods as branch and bound [7] and Lagrangian dual relaxation [3] can be used to solve the p-median problem.

In an inverse location problem, the task is to change some parameters of the problem, such as traffic connections or weights of nodes, at the minimum cost so that a pre-specified solution becomes optimal, see [8]. Burkard et al. [4] investigated the inverse 1-median problem with variable weights of nodes and proved that the problem is solvable by a greedy type algorithm in $O(n \log n)$ time in a tree network or a plane and in $O(n^2)$ time on cycles. The well-known origin-destination $(O-D)$ trip matrix estimation problem is usually interpreted as the inverse of the traffic assignment problem (TAP), see [2]. Abareshi et al. [1] investigated the $O-D$ demands through a path flow estimator applying the entropy maximizing approach.

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TABLE 1. The notation used in the bi-level model

Notation	Definition
N	$= \{v_1, ..., v_n\}$ The set of nodes
E	The set of links
I	The index set of client nodes
J	The index set of existing facilities
$\bar{w}_j$	The observed (target) amount of provided demand by facility $v_j, j \in J$
w_j	The decision variable representing the estimated amount of provided demand by facility $v_j, j \in J$
$\bar{s}_i$	The target demand of node $v_i, i \in I$
s_i	The decision variable representing the estimated demand of node $v_i, i \in I$
x_{ij}	The decision variable representing the amount of demand of node $v_i, i \in I$ provided by facility $v_j, j \in J$
d_{ij}	Per unit cost to serve the demand of node $v_i, i \in I$ by facility $v_j, j \in J$

In this paper, we propose a bi-objective model to estimate the demands of client nodes by using the number of visitors to each selected facility and some target values so that the sum of squared errors and the transportation cost would be minimized. The ϵ-constraint method is applied to determine weakly efficient solutions to the bi-objective problem where the efficiency of the estimated solutions is verified by Benson's method, see [5].

2. Bi-objective demand estimation p-median problem

Consider the network $G = (N, E)$, where the other frequently used notations are listed in Table (1). Giving the locations of existing facilities with the observed amount of provided demands by each one and also some target values for nodes' demands, the problem formulation to estimate the true values of demands is proposed as follows:

$$BODEPM : \min_x \left\{ \begin{array}{ll} f_1(x) = & \sum_{j\in J}(w_j - \bar{w}_j)^2 + \sum_{i\in I}(s_i - \bar{s}_i)^2 \\ f_2(x) = & \sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij} \end{array} \right. \tag{2.1}$$

$$s.t \sum_{i\in I} x_{ij} = w_j, \ \forall j \in J \tag{2.2}$$

$$\sum_{j\in J} x_{ij} = s_i, \ \forall i \in I \tag{2.3}$$

$$x_{ij} \geq 0 \ \forall i \in I, j \in J,$$

where the objective functions $f_1(x)$ and $f_2(x)$ respectively minimize the total sum of squared errors between the estimated and the target amounts together with the transportation cost. The constraints (2.2) and (2.3) estimate the nodes' demands and provided customers by facilities.

Definition 2.1. A feasible solution $\hat{x} \in X$ is called efficient or Pareto optimal for a bi-objective problem $\min_{x\in X} f(x) = (f_1(x), f_2(x))$, if there is no other feasible solution $x \in X$ such that $f_i(x) \leq f_i(\hat{x}),\ i = 1, 2$ and $f(x) \neq f(\hat{x})$. If the inequality is stated strictly, then the solution is called weakly efficient. The set of all efficient solutions $\hat{x} \in X$ is denoted by X_E and called the efficient set, [5].

Alternative approaches as the weighted sum and ϵ-constraint methods can be applied to solve multi-objective programming problems, see [5]. We use the ϵ-constrained method to obtain the weakly efficient solutions for the bi-objective problem (2.1). The approach is to minimize one objective subject to the additional constraint that the other objective is less than a threshold ϵ. The problem to be solved is introduced as one of the following models:

$$P_1 : \min f_1(x) = \sum_{j\in J}(w_j - \bar{w}_j)^2 + \sum_{i\in I}(s_i - \bar{s}_i)^2 \tag{2.4}$$
$$s.t. \ \sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij} \leq \epsilon_1.$$

$$P_2 : \min f_2(x) = \sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij} \tag{2.5}$$
$$s.t. \ \sum_{j\in J}(w_j - \bar{w}_j)^2 + \sum_{i\in I}(s_i - \bar{s}_i)^2 \leq \epsilon_2.$$

with the constraints (2.2) and (2.3) and non-negativity conditions.

Theorem 2.2. *Let $\hat{x}$ be an optimal solution of (2.4) or (2.5) for some $\epsilon = (\epsilon_1, \epsilon_2)$. Then $\hat{x}$ is weakly efficient to Problem (2.1), see [5].*

Theorem 2.3. *The feasible solution $\hat{x}$ of Problem (2.1) is efficient if and only if there exists an $\epsilon = (\epsilon_1, \epsilon_2)$ such that $\hat{x}$ is an optimal solution for both problems (2.4) and (2.5), see [5].*

Both problems (2.4) and (2.5) should be solved by applying quadratic programming algorithms such as the active set or interior point methods, see [6]. In the following, taking some values for $\epsilon = (\epsilon_1, \epsilon_2)$, different weakly efficient solutions are provided for a small network where the problems (2.4) and (2.5) are solved by *Lingo* 17's solvers. Then, considering Benson's method [5], the efficiency of the solutions is investigated.

Example 2.4. Figure (1) represents a small network with 9 nodes where the lengths of links are given next to them. The amount of d_{ij} for each pair (i, j) is considered as the shortest distance between i and j. The real values of nodes' demands, dem_is, are inserted in Table (2) where considering these values, the ordinary p-median problem is solved for $p = 3$. Then, using the selected facilities as well as the provided demands by each one, $\bar{w}_j$s, the weakly efficient solutions for Problem (2.1) are obtained via solving the problems (2.4) and (2.5) by selecting different vectors $\epsilon = (\epsilon_1, \epsilon_2)$ where the values of target demands, $\bar{s}_i$s, and their estimated quantities, s_is, as well as the target amounts of provided demands by selected facilities, $\bar{w}_j$s, are written in Table (2).

Applying Benson's method, it is revealed that all the obtained solutions written in Table (2) are effiecient. In addition, in all cases, the estimated values of provided demands by selected facilities, w_js, are equal to their target amounts, $\bar{w}_j$s, which have been deleted from Table (2). As Table (2) illustrates, the amounts of nodes' demands, s_is, determined by Problem (2.5) are generally more accurate estimates for their real values, dem_i; however,

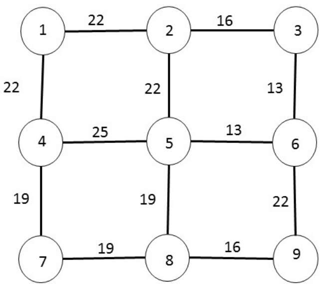

FIGURE 1. The grid network with 9 nodes

TABLE 2. Data to the network

$\epsilon_1 = 7000$, Problem (2.4)										$\epsilon_2 = 1000$, Problem (2.5)									
$j\backslash i$	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	$\bar{w}$
2	46.5	113.5	0	0	0	0	0	0	0	49.3	110.7	0	0	0	0	0	0	0	160
6	0	0	76	0	66	91.9	0	6.3	49.9	0	0	74.7	0	64.7	87.4	0	12.1	51	290
7	0	0	0	92.7	0	0	121	56.4	0	0	0	0	96.2	0	0	119.7	54.1	0	270
s	46.5	113.5	76	92.7	66	91.9	121	62.7	49.9	49.3	110.7	74.7	96.2	64.7	87.4	119.7	66.2	51	
$f_1(x) = 1574.310,\ f_2(x) = 7000$										$f_1(x) = 1000,\ f_2(x) = 7260.864$									
$\epsilon_1 = 5000$, Problem (2.4)										$\epsilon_2 = 500$, Problem (2.5)									
2	17.6	142.4	0	0	0	0	0	0	0	52.4	107.6	0	0	0	0	0	0	0	160
6	0	0	74.9	0	64.9	125	0	0	25.2	0	0	73.4	0	63.4	82.3	0	18.8	52.1	290
7	0	0	0	73.9	0	0	152.1	43.9	0	0	0	0	100.2	0	0	118.4	51.4	0	270
s	17.6	142.3	74.9	73.9	64.9	125	152.1	43.9	25.2	52.4	107.6	73.4	100.2	63.4	82.3	118.4	70.2	52.1	
dem	50	110	80	100	70	80	100	70	60	50	110	80	100	70	80	100	70	60	
$\bar{s}$	60	100	70	110	60	70	115	80	55	60	100	70	110	60	70	115	80	55	
$f_1(x) = 11525.26,\ f_2(x) = 5000$										$f_1(x) = 500,\ f_2(x) = 7560.826$									

decreasing the levels of ϵ_1 and ϵ_2 results in the rising of the values $f_1(x)$ and $f_2(x)$ in the problems (2.4) and (2.5), respectively.

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