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OPTIMAL CONTROL OF A MEN'S INFERTILITY MODEL

SOMAYEH GHIASI HAFEZI[1]*, SOHRAB EFFATI[2], AND ZAHRA ESLAMI GISKI[1]

[1] *Young Researchers and Elite Club , Islamic Azad University, Sirjan, Iran*
somayehghiyasi@gmail.com; Z.eslami@iausirjan.ac.ir

[2] *Department of Faculty of Mathematic Ferdowsi University of Mashhad(FUM).*
s-effati@um.ac.ir

Abstract. In this paper, we consider a mathematical model of a men's infertility model with azoospermia and two control variables. A deterministic system of differential equation is presented. There are two treatment methods available in the infertility Clinic to an infertile individual. One is drug treatment, and the other is IVF treatment. We intend to control the infertile individuals with using control terms drug and IVF treatment strategies. We aim to minimize the total number of infertile individuals and the cost associated with the use of educational campaign and treatment on interval time. We used Pontryagin's maximum principle to characterize the optimal levels of the two controls. The resulting optimality system is solved numerically. The results show that the optimal combination of treatment and educational campaign strategy required to achieve the set objective will depend on the relative cost of each of the control measures. The results from our simulation is discussed.

1. Introduction

In the past, women were the focus of attention regarding the couples infertility, and male factors were considered uncommon. Nowadays, the poor quality of sperm accounts in 20 percent of infertile couples suffering from infertility, is an important interfering factor for men's infertility.[1]

In the present study, a mechanical model for describing couples' infertility focusing on men's cause is considered. To do so, all infertile couples in time t with $N(t)$ are shown. In this model, the population is divided into six groups, including:

i) The susceptible couples include those couples who have just referred to the clinic but

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* Speaker.

whose illness has not been confirmed.
ii) Suffering couples are those who have visited the doctor and taken preliminary tests and the cause of infertility has been diagnosed as being related to men.
iii) People who are under treatment using drug.
iv) People who have received IVF treatment.
v) People who have undergone testicle biopsy using ICSI.
vi) Couples who have received treatment and become fertile.
These groups (i−iv) are shown as S, I, T_1, T_2, T_3, R, respectively. So, the model is called men's infertility.

2. Model Formulation

Besides symbols introduced in the previous section, it is assumed that the size of all people who have referred to the clinic in time t equals $N(t)$. So, we have

$$N(t) = S(t) + I(t) + T_1(t) + T_2(t) + T_3(t) + R(t).$$

assume α and γ are the rate of death (the rate of patients who left the clinic without any results or die naturally) and birth (the rate of patients who have referred to the clinic), respectively. The rate of the return of the illness in each couple with men's cause is β. The rate of treatment in people chosen for drug treatment in time unit is shown by δ_1 and the rate of treatment for people chosen for IVF treatment in time unit is shown by δ_2. Considering the fact that $N(t)$ comprises the number of infertile couples in time t, the average number of men who have returned to illness in time t will be shown by $\beta N(t)$. The possibility of illness in a vulnerable man equals $\frac{S(t)}{N(t)}$. Therefor, the number of new patients in time unit equals $\beta N(t)\frac{S(t)}{N(t)}$ [2]. People who have the possibility of returning to illness in time t are $(\delta_1 T_1(t) + \delta_2 T_2(t))$. Thus, the number of new patients equals

$$\beta N(t)\frac{S(t)}{N(t)}(\delta_1 T_1(t) + \delta_2 T_2(t)) = \beta S(\delta_1 T_1(t) + \delta_2 T_2(t)).$$

Based on the following information we have

$$\begin{cases} \dot{S}(t) = \gamma N(t) - \alpha S(t) - \beta(\delta_1 T_1(t) + \delta_2 T_2(t))S(t) \\ \dot{I}(t) = \beta(\delta_1 T_1(t) + \delta_2 T_2(t))S(t) - (\alpha + \lambda)I(t) \\ \dot{T_1}(t) = v\lambda I(t) - (\alpha + \delta_1)T_1(t) \\ \dot{T_2}(t) = k(t)h(t)\delta_1 T_1(t) - (\alpha + \delta_2)T_2(t) \\ \dot{T_3}(t) = (1 - k(t))h(t)\delta_1 T_1(t) + fr\delta_2 T_2(t) - (\alpha + \delta_3)T_3(t) \\ \dot{R}(t) = (1 - h(t))\delta_1 T_1(t) + (1 - v)\lambda I + (1 - r)\delta_2 T_2(t) + u_3\delta_3 T_3(t) - \alpha R(t) \\ \dot{N}(t) = (\gamma - \alpha)N(t) - (1 - f)r\delta_2 T_2(t) - (1 - u_3)\delta_3 T_3(t) \end{cases} \tag{2.1}$$

The rate of vulnerable people in time t equals $\gamma N(t)$. The number of vulnerable people who have left the clinic equals $\alpha S(t)$. Group D consists of the people who have left the clinic due to failure in treatment methods, old age, menopause and Azoospermia (see Figure 1a). The rate of medical treatment in group $I(t)$ is λ and the rate of recovery in group $\lambda I(t)$ is

$(1-v)$. Therefor v is a fraction of $\lambda I(t)$ who have received drug treatment. The number of people who enter group D is $\alpha I(t)$ (see Figure 1b). A percentage of $\delta_1 T_1(t)$ who have received drug and have recovered in time t is shown by $(1-h)$ and h is a fraction of $h\delta_1 T_1(t)$ who have received IVF. K is a percentage of $h\delta_1 T_1(t)$ who have entered $T_2(t)$ treatment and a percentage of $h\delta_1 T_1(t)$ who have entered T_3 treatment is shown by $(1-k)$. The number of people entering group D equals $\alpha T_1(t)$ (see Figure 1c). The rate of recovery in $\delta_2 T_2(t)$ is $(1-r)$. A fraction of $\delta_2 T_2(t)$ who have got no result from IVF and hence, are under ICSI treatment is r. f is a percentage of $r\delta_2 T_2(t)$ who have entered $T_3(t)$ treatment and a fraction of $r\delta_2 T_2(t)$ who have left the clinic because of failure in treatment is shown by $(1-f)$. The number of people entering group D equals $\alpha T_2(t)$ (see Figure 1d). The rate of recovery for people using testicle biopsy, TEST who have been chosen using ICSI in time unit is shown by δ_3. The percentage of $\delta_3 T_3(t)$, who have received ICSI treatment and have recovered in time t is u_3. A fraction of $\delta_3 T_3(t)$, who have not become fertile due to menopause, Azoospermia or old age and have left the treatment is $(1-u_3)$. The number of people entering group D equals $\alpha T_3(t)$ (see Figure 1e). The number of people entering group D equals $\alpha R(t)$ (see Figure 1f). The rate of $(1-f)r\delta_2 T_2(t)$ and $(1-u_3)\delta_3 T_3(t)$ are the population who have not become fertile because of treatment failure, menopause or old age ad thus, have left the clinic. The number of people entering group D is $\alpha N(t)$.

Based on the given information we have the following figures:

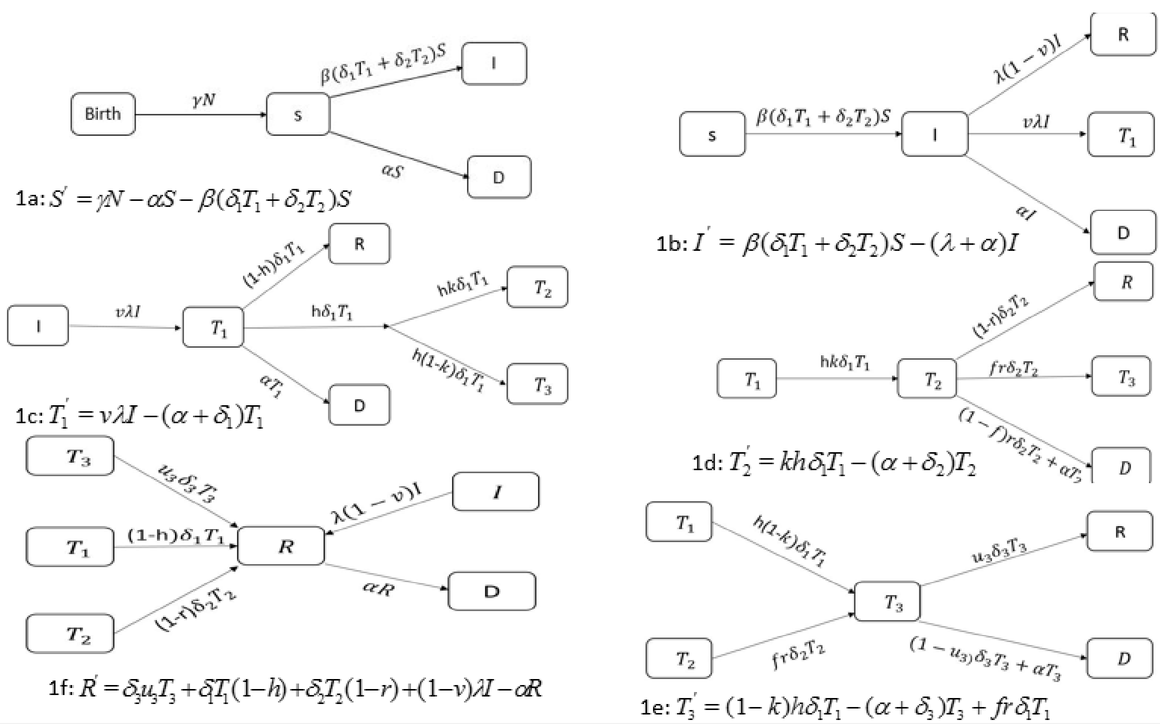

FIGURE 1. All figures of men's infertile

3. The optimal control problem

In the next section we deafne the objective function as in [4], to minimize costs at the rates of IVF treatment and Drug treatment.

Pontryagin minimum principle: Pontryagin's minimum (or maximum) principle is used in optimal control theory to find the best possible control for taking a dynamical system from one state to another, especially in the presence of constraints for the state or input controls [3].

Our problem is to minimize the objective functional

$$\min_{(h(.),k(.))\in U}\left\{\int_{t_0}^{t_f}\frac{1}{2}(h(t)^2+k(t)^2-R^2(t))dt\right\} \tag{3.1}$$

where $h(t), k(t)$ are measurable function such that the control constraint

$$U=\{(h(t),k(t)) \mid 0\leq h(t)\leq 1, 0\leq k(t)\leq 1, t\in[t_0,t_f]\}.$$

and y(t) is the solution of system 2.1 The aim is to find an optimal control for minimizing the objective functional defined in 2.1. The system seeks to minimize the aggregate cost in (3.1) by appropriately regulating $h(.)$ and $k(.)$ at all t subject to (2.1).[3]

Because the optimal control problem (2.1) is a constrained control problem, we use the Pontryagin's Maximum Principle. Hence, we give the minimized pointwise Hamiltonian as follows. This principle converts the optimal control problem (2.1) into a problem minimizing pointwise a Hamiltonian H,with respect to h and k.

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