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AN APPROXIMATION FOR DELAY OPTIMAL CONTROL OF NONLINEAR SYSTEMS

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Abstract. A successive approximation approach to obtain the solution of the optimal control of nonlinear time-delay systems with equal delay in state and control vectors is given in this brief. By employing the approximation approach, the original problem is transformed into a sequence of inhomogeneous linear two-point boundary value problem. Applying the finite-step iteration of nonlinear compensation sequence, we can obtain feed-forward and feedback suboptimal control laws. A simulation example is presented to substantiate the obtained results.

1. Introduction

The control and state estimation problems in presence of input or measurement delays have received growing attention due to its relevance in many emerging applications such as network control systems, where delays must be taken into account in the transmission of input signals; see [1, 2]. The time-delay system is therefore a very important class of systems, so control and optimization of time delay systems have been of interest to many investigators; see [3, 4].

2. Problem formulation and optimality conditions

In this section, the problem formulation and the optimality conditions of the problem are stated.

Key words and phrases. Nonlinear time-delay systems, optimal control, approximation approach.

* Speaker.

Consider the nonlinear time-delay system as the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1x(t-\tau) + Bu(t) + B_1u(t-\tau) + f(x), && t_0 \le t \le t_f, \\ x(t) &= \phi(t), && t_0 - \tau \le t \le t_0, \\ u(t) &= \theta(t), && t_0 - \tau \le t \le t_0, \end{aligned} \tag{2.1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are, respectively, the state and the control vectors, A, A_1, B, and B_1 are real constant matrices of appropriate dimensions, and $\tau > 0$ is the time-delay. Also, $\phi(\cdot)$ and $\theta(\cdot)$ are the initial state and control vector functions and $f : \mathbb{R}^n \to \mathbb{R}^n$. Throughout the paper, we assume that $f(\cdot)$ is differentiable and satisfies uniformly Lipschitz conditions. The finite-time quadratic performance index (cost functional) is given by

$$J(u) = \frac{1}{2}x^T(t_f)Q_f x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} x^T(t)Qx(t) + u^T(t)Ru(t)dt, \tag{2.2}$$

where $Q, Q_f \in \mathbb{R}^{n\times n}$ are positive semidefinite matrices and $R \in \mathbb{R}^{m\times m}$ is a positive definite matrix. The objective is to minimize J by designing an optimal control law u^*.

3. Successive approximation process

This section introduces a successive approximation approach to solve the TPBV problem. First, consider the following nonlinear time-delay system:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + A_1(t)x(t-\tau) + f(x), && t \in [t_0, t_f], \\ x(t) &= \phi(t), && t_0 - \tau \le t \le t_0, \end{aligned} \tag{3.1}$$

where $x \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$, $\phi(t)$ is the initial state vector function, and $f : \mathbb{R}^n \to \mathbb{R}^n$ with $f(0) \equiv 0$ has the uniformly Lipschitz property. The general solution of the nonlinear time-delay system (3.1) has the following form:

$$x(t) = \Phi(t, t_0)\phi(t_0) + \int_{t_0}^{t} \Phi(t, r)[A_1(r)x(r-\tau) + f(x(r))]dr.$$

Now, we define the sequence $\{x^{(k)}\}$ as follows:

$$\begin{aligned} x^{(k)}(t) &= \Phi(t, t_0)\phi(t_0) + \int_{t_0}^{t}[\Phi(t, r)f(x^{(k-1)}(r))]dr + \int_{t_0}^{t}[\Psi(t, r)x^{(k-1)}(r-\tau))]dr, \quad t \in [t_0, t_f], \\ x^{(k)}(t) &= \phi(t), \qquad t_0 - \tau \le t \le t_0, \qquad k = 1, 2, \ldots, \end{aligned} \tag{3.2}$$

where $\Psi(t, r) = \Phi(t, r)A_1(r)$ and $\Phi(\cdot, \cdot)$ is the state transfer matrix with respect to the matrix $A(t)$. Also $x^{(0)}(t) = \Phi(t, t_0)\phi(t_0)$. Here, the following lemma states a useful result.

Lemma 3.1. *Sequence (3.2) is uniformly convergent to the solution of system (3.1).*

The sequence of TPBV problem for solving the original problem based on the optimality conditions of the delay optimal control problems [5] can be constructed as follows:

$$\dot{x}^{(k)}(t) = \begin{cases} Ax^{(k)}(t) + A_1 x^{(k-1)}(t-\tau) - M\lambda^{(k)}(t) - N\lambda^{(k-1)}(t+\tau) - B_1\theta(t-\tau) + f(x^{(k-1)}), \\ \qquad\qquad t_0 < t \le t_0 + \tau, \\ Ax^{(k)}(t) + A_1 x^{(k-1)}(t-\tau) - (M+S)\lambda^{(k)}(t) - N\lambda^{(k-1)}(t+\tau) - T\lambda^{(k-1)}(t-\tau) + f(x^{(k-1)}), \\ \qquad\qquad t_0 + \tau < t \le t_f - \tau, \\ Ax^{(k)}(t) + A_1 x^{(k-1)}(t-\tau) - (M+S)\lambda^{(k)}(t) - T\lambda^{(k-1)}(t-\tau) + f(x^{(k-1)}), \\ \qquad\qquad t_f - \tau < t \le t_f. \end{cases} \tag{3.3}$$

$$-\dot{\lambda}^{(k)}(t) = \begin{cases} Qx^{(k)}(t) + A^T\lambda^{(k)}(t) + f^T_{x^{(k-1)}}(x^{(k-1)})\lambda^{(k-1)}(t) + A_1^T\lambda^{(k-1)}(t+\tau), & t_0 \le t \le t_f - \tau, \\ Qx^{(k)}(t) + A^T\lambda^{(k)}(t) + f^T_{x^{(k-1)}}(x^{(k-1)})\lambda^{(k-1)}(t), & t_f - \tau \le t \le t_f, \end{cases} \tag{3.4}$$

$$x^{(k)}(t) = \phi(t), \qquad t_0 - \tau \le t \le t_0, \tag{3.5}$$

$$\lambda^{(k)}(t_f) = Q_f x^{(k)}(t_f), \tag{3.6}$$

$$\lambda^{(0)}(t) = 0, \qquad x^{(0)}(t) = x(0), \qquad t_0 \le t \le t_f, \tag{3.7}$$

for $k = 1, 2, \dots$ and the delay τ has to be chosen such that $t_0 + \tau < t \le t_f - \tau$. The control sequence is also given by

$$u^{(k)}(t) = \begin{cases} -R^{-1}(B^T\lambda^{(k)}(t) + B_1^T\lambda^{(k-1)}(t+\tau)), & t_0 < t \le t_f - \tau, \\ -R^{-1}B^T\lambda^{(k)}(t), & t_f - \tau < t \le t_f, \end{cases} \tag{3.8}$$

$$u^{(k)}(t) = \theta(t), \qquad t_0 - \tau \le t \le t_0. \tag{3.9}$$

Theorem 3.2. *Suppose that $x^{(k)}(t)$ and $u^{(k)}(t)$ satisfy the sequences of (3.3)–(3.4) and (3.8)–(3.9), respectively. Then, $x^{(k)}(t)$ and $u^{(k)}(t)$ converge uniformly to the optimal trajectory $x^*(t)$ and the optimal control law $u^*(t)$ for system (2.1) with respect to the quadratic cost functional (2.2), respectively.*

4. An example

Example 4.1. Consider the second order nonlinear time-delay system (2.1), where

$$A = \begin{bmatrix} 0 & 0 \\ 1 & -\sqrt{2} \end{bmatrix}, \qquad B = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix},$$

$$f(x) = \begin{bmatrix} x_2^2 & x_1x_2 \end{bmatrix}^T, \qquad \tau = 0.5,$$

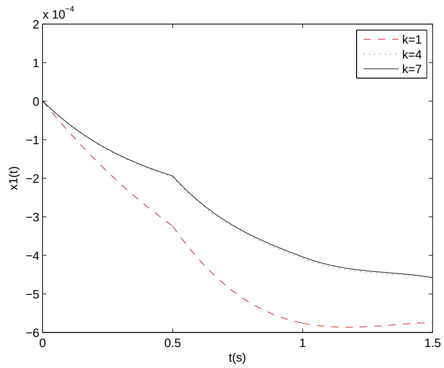
$$x(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \qquad -\tau \le t \le 0,$$

$$u(t) = 0, \qquad -\tau \le t \le 0.$$

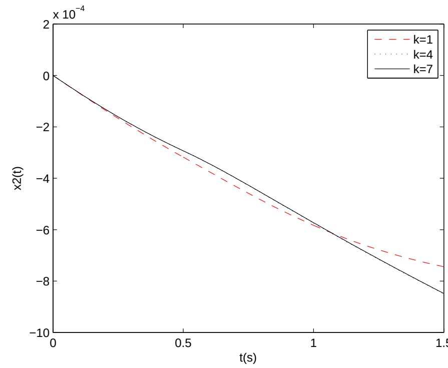
As well as quadratic cost function is as follows:

$$J = (\frac{1}{2}) \int_0^{1.5} \{x^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x + u^2\} dt.$$

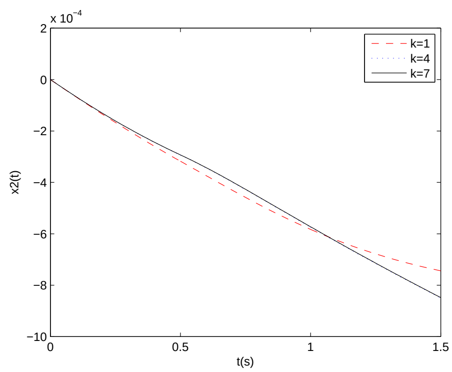
The trajectories of states and control are shown in Figures 1(A), 1(B), and 1(C), respectively.



(A) Trajectories of the state $x_1(t)$ in Example 4.1.



(B) Trajectories of the state $x_2(t)$ in Example 4.1.



(C) Trajectories of the control $u(t)$ in Example 4.1.

Figure 1. Trajectories behaviour in Example 4.1.

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