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A DESCENT APPROACH WITHOUT USING UNKNOWN PARAMETERS FOR SOLVING NONSMOOTH OPTIMIZATION PROBLEMS

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Abstract. A neural network model is proposed for solving some class of nonsmooth optimization problems. The model is based on steepest descent approach and it is formulated by a differential inclusion and is implemented by circuits. Under suitable assumptions, trajectories converge to a point optimal solution set. In this model, there is not any unknown parameter, which is its major difference in comparision with similar models.

1. Introduction

Neural network models as a parallel approach for solving mathematical problems are tools for solving real time problems in reasonable time (see [1]-[5] and references therein). In this paper, we introduce a recurrent neural network for solving a class of nonsmooth optimization problems, with inequality constraints. Our differential inclusion-based model does not use any penalty parameter. We prove that solution trajectories globally converge to an optimal solution of the problem. For differentiable problems, we represent the circuit diagram of the designed neural network. We solve three numerical examples to confirm the effectiveness of the theoretical results. The first one is a nonsmooth test problem and the second one is finding minimum evolution time problem which arises in applications such as markov decision processes. We solve these problems via penalty-based model, with different values of penalty parameters and compare our model with this one. Finally, to illustrate the effectiveness of the new model to solve nonconvex problems, we apply our new model to solve a a problem whose objective function is nonconvex. Consider the constraint

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optimization problem

$$\begin{array}{ll} \min & f(x) \\ s.t & g_i(x) \leq 0,\ i = 1, 2, \cdots, m, \end{array} \tag{1.1}$$

where $g_i : \mathbb{R}^n \to \mathbb{R}$ for each $i = 1, 2, \cdots, m$ and $f : \mathbb{R}^n \to \mathbb{R}$ are nonsmooth real valued functions. Furthermore, we assume that problem (1.1) is feasible.

2. Methodology

Consider the following general differential inclusion:

$$\begin{array}{l} \dot{x}(t) \in F(x(t)), \\ x(0) = x_0, \end{array} \tag{2.1}$$

where $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is an upper semicontinuous set valued map, with nonempty compact and convex values. Furthermore, consider energy function

$$V(x, y) : \mathbb{R}^n \times \Omega^* \to [0, \infty).$$

Assume the following conditions hold:
Condition 1: $V(x, y)$ is continuously differentiable with respect to variable x and continuous with respect to variable y.

Condition 2: If $\|x\| \to \infty$, then for each $x^* \in \Omega^*$, $V(x, x^*) \to \infty$.

Condition 3: For each $x^* \in \Omega^*$ and $x \in \mathbb{R}^n$, if $V(x, x^*) = 0$, then $x \in \Omega^*$. In addition, if $x = x^*$, then, $V(x, x^*)$=0.

Condition 4: For any $x^* \in \Omega^*$ and each $x \in \mathbb{R}^n$, the following inequality holds:

$$F^T(x)\frac{dV(x, x^*)}{dx} \leq 0.$$

Equality holds only if $x \in \Omega^*$.

Condition 5: There exists $x^{**} \in \Omega^*$, such that for any compact set $C \subset \mathbb{R}^n$, that $C \cap \Omega^* = \emptyset$, there exixts $\delta_c > 0$, which satisfies the following inequality:

$$F^T(x)\frac{dV(x, x^{**})}{dx} < -\delta_C.$$

Theorem 2.1. *Suppose Ω^* is bounded and conditions 1-5 hold. Then for any initial solution $x_0 \in \mathbb{R}^n$, solution trajectory path of differential inclusion (2.1) converges to an element of Ω^*.*

Remark 2.2. Note that Ω^* in Theorem 2.1 can be assumed to be any other compact subset of Ω. For example, if $\partial\Omega$ is bounded, then it is compact, and we can use $\partial\Omega$ instead of Ω^* in the theorem.

3. The New Model

Now we design a new differential inclusion-based neural network to solve problem (1.1). Assume that objective functions and constraints are locally Lipschitz. Define the following activation function:

$$\theta[u] = \begin{cases} 1, & u > 0, \\ [0,1]\,, & u = 0, \\ 0, & u < 0. \end{cases} \tag{3.1}$$

Consider the following differential inclusion

$$\dot{x}(t) \in -\left\{ \prod_{i=1}^{m} (1 - \theta\left[g_i(x(t))\right]) \right\} \partial f(x(t)) - \sum_{i=1}^{m} \theta\left[g_i(x(t))\right] \partial g_i(x(t)). \tag{3.2}$$

Remark 3.1. for differentiable functions f and g_is, $i = 1, 2, \cdots, m$, differential inclusion (3.2), can be implement via circuit form. Figure 1, shows block diagram, corresponding to recurrent neural network (3.2),

Now, we show that right-hand side of differential inclusion (3.2) is an upper semicontinuous set valued map and, consequently, there exists a local solution trajectory for such a differential inclusion. Assume that the following assumption holds for problem (1.1):
Assumption 1: f is convex over Ω and can be nonconvex on $\mathbb{R}^n$. Constraint functions g_is are convex and Slater condition holds; interior set of the feasible region is nonempty.

Theorem 3.2. *Suppose that f is convex over Ω and $g_i, i = 1, 2, \cdots, m$ are convex over $\mathbb{R}^n$, then any solution trajectory of differential inclusion (3.2) is globally convergent to an optimal solution of the problem (1.1).*

References

[1] A. Hosseini, S. M. Hosseini, A new steepest descent differential inclusion-based method for solving general nonsmooth convex optimization problems , J. Optim.Theory Appl. **159** (698-720) 2013.

[2] A. Hosseini, S. M. Hosseini, M. Soleimani-damaneh, A differential inclusion-based approach for solving nonsmooth convex optimization problems, Optimization, **62** (1203-1226) 2013.

[3] A. Hosseini, J. Wang, S. M. Hosseini, A recurrent neural network for solving a class of generalized convex optimization problems, IEEE Trans. Neural Netw. **17** (1500-1510) 2006.

[4] A. Hosseini, A non-penalty recurrent neural network for solving a class of constrained optimization problems, Neural Netw. **73** (10-25) 2016.

[5] X. Hu and J. Wang, Solving pseudomonotone variational inequalities and pseudoconvex optimization problems using the projection neural network, Neural Netw. **44** (78-86) 2013.

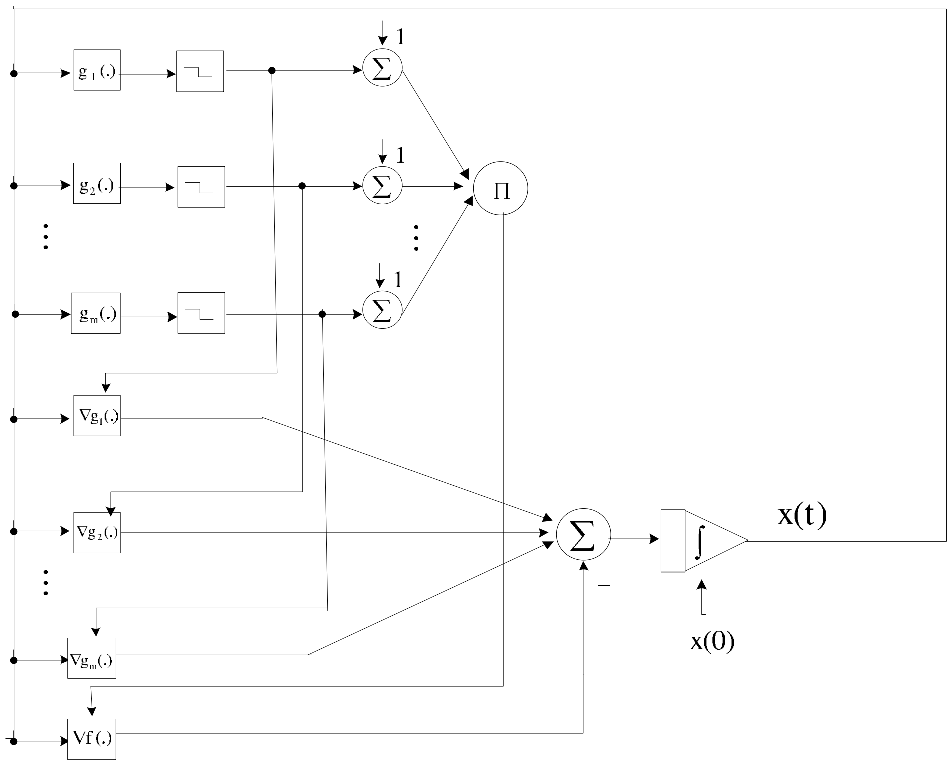

FIGURE 1. Block diagram of the recurrent neural network in (3.2). The step function is $-\theta(.)$ (θ is defined in equation (3.1))